YTU Physics Department, 2016-2017 Fall Semester		Exam Date: 05 November 2016				Exam Time: 100 min.		
FIZ1001 Physics-1 Midterm-I		P1	P2	P3	P4	P5	P6	TOTAL
Name Surname								
Registration No								
Department								

Group No	Exam Hall	Signature of the Student
Lecturer's Name Surname		

The 9th article of Student Disciplinary Regulations of YÖK Law No.2547 states ***"Cheating or helping to cheat or attempt to cheat in exams"*** de facto perpetrators **takes one or two semesters suspension** penalty. Calculators are not allowed. Do not ask any questions about the problems. There will be no explanations. Use the allocated areas for your answers and write legible.

PROBLEM 1 (12p)

The velocity of a particle moving in the xy-plane is given by the velocity vector $\vec{v} = 2t\hat{\imath} - 3t^2\hat{\jmath}$ (m/s).
The particle is at the origin at $t = 0$.

a) Find the position of the particle as a function of time.

$d\vec{r} = \vec{v}\,dt$ 1 $\int_0^{\vec{r}} d\vec{r} = \int_0^t (2t\hat{\imath} - 3t^2\hat{\jmath})dt$

2 $\vec{r} = t^2\hat{\imath} - t^3\hat{\jmath}$ (m)

b) Find the total acceleration ($\vec{a}$) of the particle.

1 1

$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{\imath} - 6t\hat{\jmath}$ (m/s²)

c) Find the magnitude of the tangential acceleration (a_t) at $t = 1s$.

$a_t = \frac{dv}{dt}$ 1 $v = (4t^2 + 9t^4)^{1/2}$

$a_t = \frac{4t + 18t^3}{(4t^2+9t^4)^{1/2}} = \frac{4 + 18t^2}{(4+9t^2)^{1/2}}$ 2

$t = 1s$ 1 $a_t = \frac{22}{\sqrt{13}}$ (m/s²)

d) Find the power transferred to the mass m=1kg at t=1s.

$\vec{F} = m\vec{a} = 2\hat{\imath} - 6\hat{\jmath}$ (N)

$\vec{v} = 2\hat{\imath} - 3\hat{\jmath}$ (m/s)

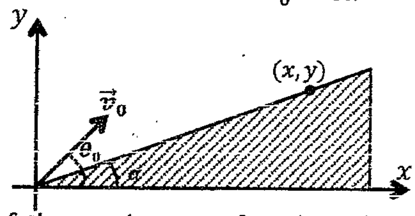
2

$P = \vec{F}\cdot\vec{v} = (2\hat{\imath} - 6\hat{\jmath})\cdot(2\hat{\imath} - 3\hat{\jmath})$

$= 4 + 18 = 22$ W 1

PROBLEM 2 (13p)

A projectile is fired uphill with a speed of $v_0 = 10$ (m/s) and a horizontal angle of θ_o, over an incline which slopes at an angle of α to the horizontal as shown. $Cos\theta_0 = 0.7$
$Sin\theta_0 = 0.7$
$Cos\alpha = 0.8$
$Sin\alpha = 0.6$
$g = 10\ m/s^2$



a) Write components of the ~~motion~~ position as a function of time $x(t)$ and $y(t)$.

$x = (v_0\cos\theta_0)t = 7t$ (m) 2

$y = (v_0\sin\theta_0)t - \frac{1}{2}gt^2 = 7t - 5t^2$ (m) 2

b) Express y as a function of x only ($y = f(x)$) for the projectile.

$t = \frac{x}{7}$ $y = x - \frac{5}{49}x^2$ 2

c) Find the position (x, y) where the projectile hits the incline.

for incline $y = (tg\alpha)x = \frac{6}{8}x = \frac{3}{4}x$ 2

$y_{inc} = y_{proc}$

$\frac{3}{4}x = x - \frac{5}{49}x^2$

$x = 49/20$ (m) 1

$y = \frac{3}{4}x = \frac{3}{4}\frac{49}{20}$

1 $y = 147/80$ (m)

d) Determine the time taken by the projectile to hit the incline.

$x = 7t$ $\frac{49}{20} = 7t$

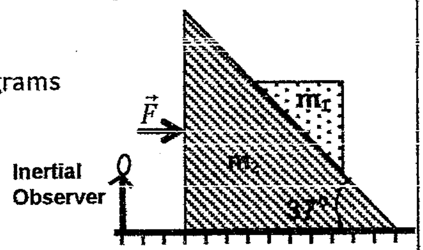
$t = \frac{7}{20}$ (s) 3

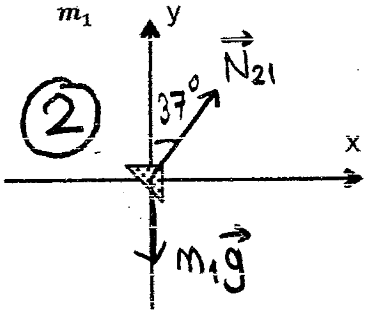
PROBLEM 3 (25p)

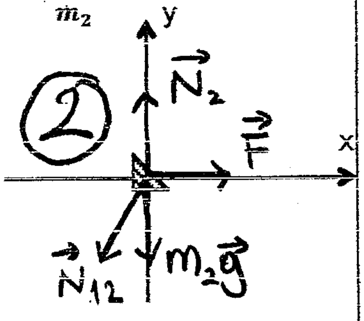
In the system of triangle shaped blocks illustrated in the figures, a constant external force $\vec{F}$ is applied in such a way that m_1 stays stationary relative to m_2. The whole system is frictionless.
Here, $m_1 = 2.4$ kg, $m_2 = 4.0$ kg, $cos37^o = 0.8$, $sin37^o = 0.6$ and $g = 10\,\frac{m}{s^2}$.

a) For an inertial observer standing still on the ground:

a-1) Draw free body diagrams







a-2) Write the equation of motions for:

m_1:

$N_{21}\sin 37 = m_1 a$ (2)

$N_{21}\cos 37 - m_1 g = 0$ (1)

m_2:

$F - N_{12}\sin 37 = m_2 a$ (2)

$N_2 - N_{12}\cos 37 - m_2 g = 0$ (1)

a-3) Find the accelerations of the masses.

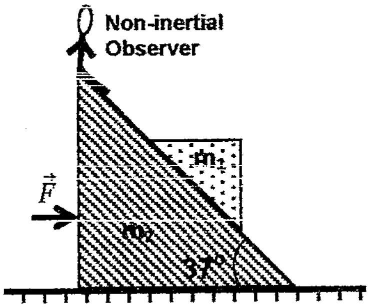
$a = g\dfrac{\sin 37}{\cos 37} = \dfrac{15}{2} = 7.5\ (m/s^2)$ (2)

a-4) Find the force F.

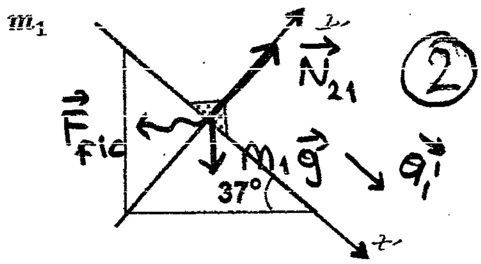
$F = (m_1 + m_2)a$

$F = 48\,N$ (2)

b) Now, $\vec{F}$ is reduced in such a way that m_2 has an acceleration of $A = 5\,m/s^2$ relative to the ground (inertial observer), and m_1 has an acceleration of a_1' relative to non-inertial observer (observer on m_2).



b-1) Draw free body diagram of m_1 according to non-inertial observer.



b-2) Write the equation of motion for m_1:

$m_1 g\sin 37 - F_{fic}\cos 37 = m_1 a_1'$ (2)

$N_{21} - F_{fic}\sin 37 - m_1 g\cos 37 = 0$ (2)

$F_{fic} = m_1 A$ (1)

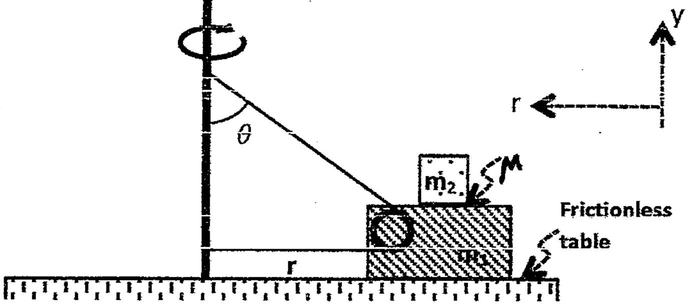
b-3) Find the acceleration a_1'.

$\cancel{m_1} g\sin 37 - \cancel{m_1} A\cos 37 = \cancel{m_1} a_1'$ (2)

$a_1' = 2\ (m/s^2)$ (2)

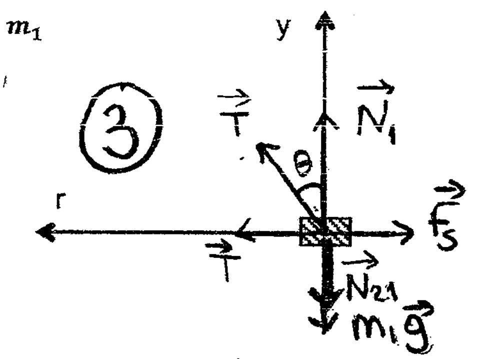
PROBLEM 4 (25p)

A block of mass m_1 is attached to a vertical rod by a single string which passes around a pulley and attached to the block as shown in figure. The block m_1 rotates on a frictionless table. Another block of mass m_2 is placed onto rough surface of mass m_1. The coefficient of static friction between the two masses is μ. The entire system rotates on a frictionless table so that the blocks are moving in a horizontal circle of r with a constant speed v. The block of mass m_2 stays stationary relative to block of mass m_1. Assume that the pulley and the string are weightless and frictionless and the

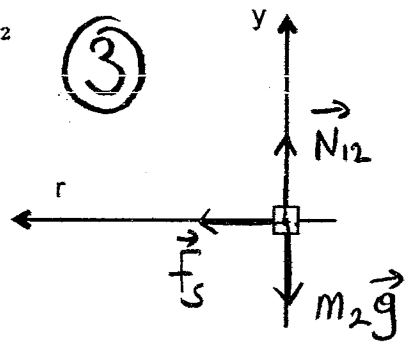


a-1) Draw the free body diagrams for m_1 and m_2.

m_1



m_2



a-2) Write the equation of motions for:

m_1:

$T\cos\theta + N_1 - N_{21} - m_1 g = 0$ 3

$T\sin\theta + T - f_s = m_1 \dfrac{v^2}{r}$ 3

m_2:

$N_{12} - m_2 g = 0$ 2

$f_s = m \dfrac{v^2}{r}$ 2

b) Find the maximum value of the speed that the mass m_2 can stay stationary relative to m_1 while entire system rotates, in terms of μ, g and r.

$f_s^{max} = \mu_s N_{12} = \mu m_2 g$ 2

$\mu m_2 g = m \dfrac{v_{max}^2}{r}$

$v_{max} = \sqrt{\mu g r}$ 2

c) Find the tension on the string while the system rotates at the maximum speed obtained in **b**, in terms of μ, θ, m_1, m_2 and g.

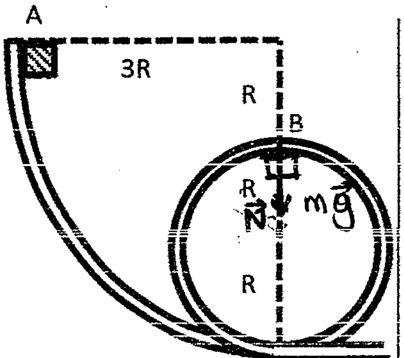
$T\sin\theta + T - f_s^{max} = m_1 \dfrac{v_{max}^2}{r}$ 2

$T(1+\sin\theta) - \mu m_2 g = m_1 \dfrac{\mu g r}{r}$

$T = \dfrac{\mu g(m_1+m_2)}{1+\sin\theta}$ 3

PROBLEM 5 (13p)

A small block of mass m starts from rest at point A and slides along a rough loop-the-loop rail as shown in the figure. A constant kinetic friction force f_k is acting on the block during its travel on the rail.



a) What is the work done between A and B by

a-1) Conservative forces

$$W_{mg} = -\Delta U = -(mgy_B - mgy_A)$$

$$W_{mg} = mgR$$ 2

a-2) Normal force

2

$$W_N = \int \vec{N} \cdot d\vec{s} = 0 \quad \text{as} \quad \vec{N} \perp d\vec{s}$$

a-3) Force of friction

$$W_{f_k} = \vec{f_k} \cdot \vec{S} = -f_k\left(\frac{2\pi 3R}{4} + \frac{2\pi R}{2}\right)$$

$$W_{f_k} = -\frac{5}{2}\pi R f_k$$ 2

b) Write equation of the conservation of energy for the block between A and B.

$$E_A + W_{f_k} = E_B$$ 1

$$K_A + U_A + W_{f_k} = K_B + U_B$$ 2

$$mg3R - \frac{5}{2}\pi R f_k = \frac{1}{2}m v_B^2 + mg2R$$

c) What should be the magnitude of the force of friction, so that the magnitude of normal force acting on the block is equal to its weight at point B.

$$N + mg = m\frac{v_B^2}{R}$$ 2

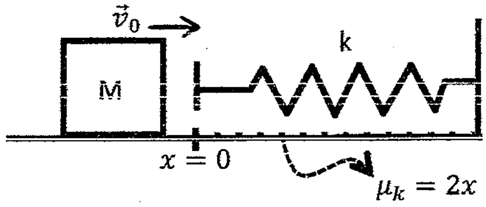
if $N = mg$ at B $\quad 2mg = m\dfrac{v_B^2}{R}$

and $\quad mg3R - \frac{5}{2}\pi R f_k = \frac{1}{2}m v_B^2 + mg2R$

$$f_k = 0$$ 2

PROBLEM 6 (12p)

A block of mass M slides along a horizontal table with speed $\vec{v}_0$. At $x = 0$, it hits a spring with spring constant k and begins to experience a friction force. The coefficient of friction is variable and is given by $\mu_k = 2x$. The block first comes momentarily to rest at $x = L$.



a) Find the work done by the forces acting on the block between $x = 0$ and $x = L$.

a-1) Gravitational force

$$W_{mg} = m\vec{g} \cdot \Delta\vec{x} = 0 \quad \text{as} \quad m\vec{g} \perp \Delta\vec{x}$$ 1

a-2) Spring force

$$W_S = -\Delta U = -\left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right)$$

$$= -\frac{1}{2}kL^2$$ 2

a-3) Normal force

$$W_N = \vec{N} \cdot \Delta\vec{x} = 0 \quad \text{as} \quad \vec{N} \perp \Delta\vec{x}$$ 1

a-4) Force of friction

$$f_k = \mu_k N = 2xN = 2xmg$$

$$W_{f_k} = -\int_0^L 2mgx\,dx = -mgx^2\Big|_0^L$$ 2

$$W_{f_k} = -mgL^2$$ 2

b) If the loss of mechanical energy, between $x = 0$ and $x = L$, due to the friction is half of the energy of the block at $x = 0$, what is the spring constant in terms of M, v_0 and L.

$$|W_{f_k}| = \frac{1}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\frac{1}{2}mv_0^2 + W_{f_k} = \frac{1}{2}kL^2$$ 2

$$\frac{1}{2}mv_0^2 - \frac{1}{4}mv_0^2 = \frac{1}{2}kL^2$$

$$k = \frac{mv_0^2}{2L^2}$$ 2