

Experiment 5

SIMPLE PENDULUM AND SPRING PENDULUM

Aims:

- i. Determination of a spring constant and observation of the period of an object in simple harmonic motion.
- ii. Determination of gravitational acceleration by simple pendulum.

Equipments: Spring, strings with different lengths, pendulum bobs (spheres) and masses, ruler, stopwatch, graphic paper, scientific calculator.

1. Introduction

Simple Harmonic Motion: A motion which is repeated with certain time intervals is called periodic motion, when an object has a periodic motion around a fixed spot, it is called oscillation.

Harmonic motion is a periodic motion in the form of sine or cosine function. When the force acting on an object having harmonic motion is balanced, its position is called equilibrium position and the distance from the equilibrium position in any time is called displacement. If the restoring force acting on the particle to equilibrium position is proportional to displacement, the motion is called simple harmonic motion.

If an object is pulled away and then released, it makes simple harmonic motion. The direction of the restoring force and the displacement are opposite, so that,

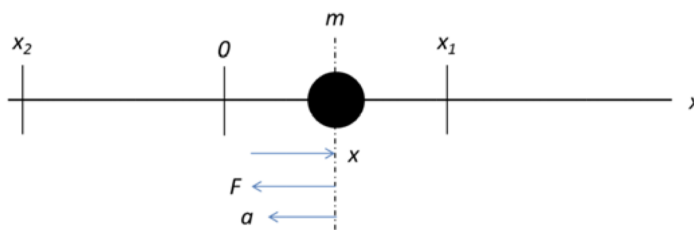


Figure 1: The object in simple harmonic motion.

$$F = -kx \quad (1)$$

where k is the constant. On the other hand, in the terms on Newton's second law, this restoring force is,

$$F = ma = m \frac{dv}{dt} = m \frac{d^2x}{dt^2} \quad (2)$$

thus,

$$-kx = m \frac{dv}{dt} \quad \text{or} \quad m \frac{d^2y}{dt^2} + kx = 0 \quad (3)$$

If, $\omega^2 = k/m$ (ω ; angular frequency) the last equation becomes,

$$\frac{d^2y}{dt^2} + \omega^2 x = 0 \quad (4)$$

Equation (4) is called harmonic oscillator equation and its solution is,

$$y = A \sin(\omega t + \delta) \quad (5)$$

where A is the amplitude and δ is the initial phase.

With the help of equation (5) we gain;

$$v = \frac{dx}{dt} = \omega A \cos(\omega t + \delta) \quad (6)$$

$$a = \frac{dv}{dt} = -\omega^2 A \sin(\omega t + \delta) = -\omega^2 y \quad (7)$$

The angular frequency can be written as, $\omega = \frac{2\pi}{T}$, then the period of the simple harmonic motion is,

$$T = 2\pi \sqrt{\frac{m}{k}} \quad (8)$$

Gravitational Acceleration: When an object is released from a certain height it falls with speeding up. As the object does not have an initial velocity, a force is necessary to start the motion. This can be explained with the laws of dynamics as it gains an acceleration. On the other hand, an object is speeding up in a free fall motion, it is obvious that it has an acceleration. This acceleration acting on the objects is called, gravitational acceleration (g), the force acting on the object is called the weight of the object. If m is the mass of the object,

$$G = mg \quad (9)$$

In other words, G is the force acting on the force by the earth and it is usually called as gravitational force. Depending on the Newton's third law, as the earth act on the object with a force of G , the object acts a force on the earth as a response.

Simple Pendulum: The system with a fixed light string carrying a mass is called simple pendulum (Figure-2). If the mass is pulled over from the equilibrium position and then

released, it will make periodic oscillations along the vertical axis with the mg gravitational force and under the tension T on the string. As shown in Figure-1, on the (x, y) plane, the component of mg along the x axis is $mg \sin \theta$, the component along the y axis is $mg \cos \theta$. Thus the tension on the string T is balanced by $mg \cos \theta$

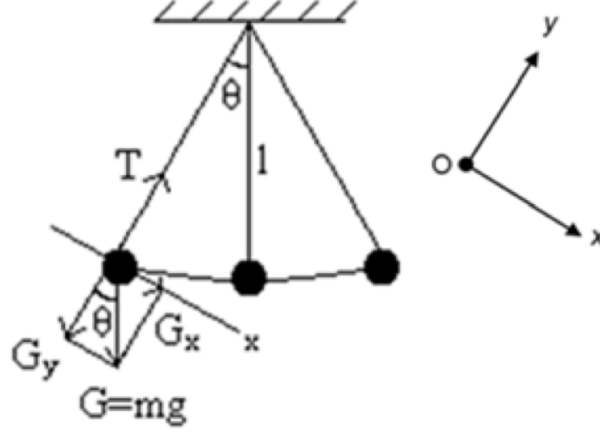


Figure 2

$mg \sin \theta$ component is the restoring force and can be written as,

$$F = mg \sin \theta \quad (10)$$

If the angle θ is small ($< 5^\circ$), $\sin \theta \approx \theta$ so, $\theta = x/\ell$. now the restoring force is,

$$F = -mg\theta = -mg\frac{x}{\ell} \quad (11)$$

Thus, the restoring force is directly proportional to displacement for small displacement values ($F \propto x$). So the simple pendulum makes simple harmonic motion. According to this, the equation below can be written,

$$F = -kx \quad (12)$$

where k is the spring constant. The $(-)$ sign in equation (12) means it is the restoring force. Using Eqs. (11) and (12),

$$-kx = -mg\frac{x}{\ell} \text{ or } k = \frac{mg}{\ell} \quad (13)$$

Using Newton's second law given by, $F = m \left(\frac{d^2x}{dt^2} \right)$, we can write,

$$-kx = m \left(\frac{d^2x}{dt^2} \right) \quad (14)$$

using $\omega^2 = \frac{k}{m}$, equation (13) becomes

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad (15)$$

This is the differential equation of simple harmonic motion. The solution of equation (15) is given below,

$$x = A \sin(\omega t + \delta) \quad (16)$$

where A is the constant amplitude value and δ is the initial phase. The solution as a function of the initial condition can be written as,

$$x = A \cos(\omega t + \delta) \quad (17)$$

On the other hand, as $\omega = \frac{2\pi}{T}$, the period of the motion is,

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{mg/\ell}} = 2\pi \sqrt{\frac{\ell}{g}} \quad (18)$$

One can understand that for small oscillations, the period of the simple pendulum is not dependent to the mass of the pendulum bob and the amplitude, it just depends on the length of the pendulum and the gravitational acceleration.

“Note that equation (17) is valid if only the angle θ is small enough.”

2. Experiment

Determination of Spring Constant

1. Hang the mass m at the end of the spring and pull it down softly from the equilibrium position and then release the system. Observe the simple harmonic motion of the system around the equilibrium position.
2. Measure the time for 10 periods to determine the period of the simple harmonic motion. Calculate the average period and write the values down in Table-1.

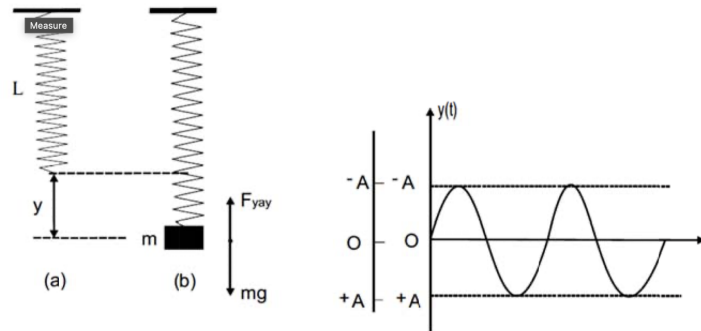
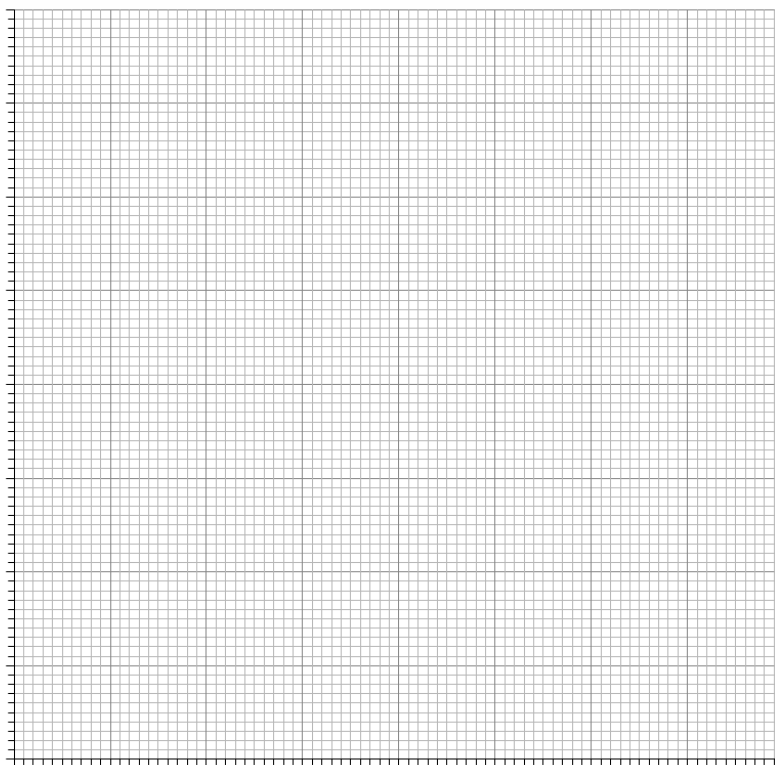


Figure 3: Spring pendulum. The spring lengthens Δy when weight m is hanged. When the spring force equals to the weight of the mass, the system is in equilibrium. If the mass is pulled down $y = A$ from the equilibrium position and then released, simple harmonic motion will be observed.

- Repeat the experiment by adding extra masses on the spring and calculate the average periods for each mass values. Write the values down in Table-1.

m (kg)	$10T$ (s)	T_{ave} (s)	T^2 (s ²)	k (N/m)

- Plot the $T^2 = f(m)$ graph using the values from Table-1. This is supposed to be a straight line passing through the origin point.



- Calculate the spring constant by equation (8) and choosing two m, T^2 pairs. Write down the results in Table 1.

Determination of Gravitational Acceleration

- Measure the length of the string from the hanging point to the sphere ℓ' , and the diameter of the sphere by a caliper and calculate the length of the pendulum $\ell = \ell' + R$. Repeat this process for the strings with 4 different lengths and write down the values in Table-2.

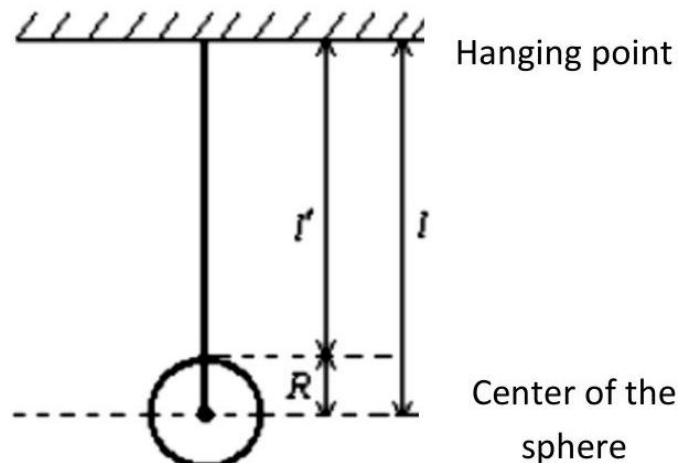


Figure 4

7. Pull over the pendulum a little (approximately 5°) and let it to oscillate. Measure the time for 10 periods of oscillation by stopwatch and find the period of the pendulum. (Note that 1 period is the time interval passing, as the pendulum reaches its starting point.)
8. Repeat the experiment bu using strings with different lengths (at least 4 times) and write down the values in Table-2.

$$2R = 10 \text{ cm} \quad R = 5 \text{ cm} \quad \ell' = \dots \quad g_{\text{theoretical}} = 9,8 \text{ m/s}^2$$

Table 1

$\ell = \ell' + R$ (m)	$10T$ (s)	T_{ave} (s)	g_{ave} (m/s ²)	$\frac{ \Delta g }{g_{\text{theoretical}}}$

9. Plot the $T^2 = f(\ell)$ graph with the help of Table-2. Calculate the gravitational acceleration g by the ℓ/T^2 ratio from the graph and equation (17).



10. Calculate the relative error on determining the gravitational acceleration comparing the theoretical value from the equation below and write down in Table-2.

$$\frac{|\Delta g|}{g_{\text{theoretical}}} = \frac{|g_{\text{theoretical}} - g_{\text{ave}}|}{g_{\text{theoretical}}} \quad (19)$$