Experiment 1
THE ANALYSIS OF AN EXPERIMENT

Purpose: To learn how to analyze experimental data and draw graphics, prediction of the results of similar experiments from the mathematical equations which are obtained from analysis, to learn how to make experimental error calculations.
Equipment and materials: Scientific calculator, pencil, eraser, ruler, graph paper.

1. Information

The heart of any experiment is making observations and measurements. Accurate measurement requires appropriate tools. When measuring a tabletop, we could use a meter stick to produce a suitable measurement. The meter stick has graduations small enough to attain a measurement to within a millimeter. One can make a measurement accurate to within a thousandth of a meter. This is good accuracy if the table is roughly a meter or longer. To use a meter stick to measure the thickness of a pencil would be inappropriate. Assuming a pencil is roughly 5 mm in diameter; one would want a tool that could give measurements accurate to a fraction of a millimeter. The vernier and micrometer calipers were developed to perform such measurements.

All measurements are subject to uncertainty; no matter how precise the instrument that is used or how careful the experiment is done. Therefore it is important to evaluate in some way the magnitude of the uncertainty in a measurement, and if possible, minimize that uncertainty.
Consider the following standard metric ruler (Figure 1).

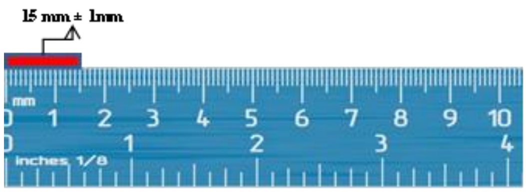


Figure 1. Demonstration of the uncertainty of the ruler.

The ruler is incremented in units of centimeters (cm). The smallest scale division is a tenth of a centimeter or 1 mm. Therefore, the uncertainty ± 1 mm. In the example above, the length of the object would be stated as 15 ± 1 mm.

The vernier caliper is an instrument that allows you measure lengths much more accurate than the metric ruler. The smallest increment in the vernier caliper you will be using is 0,1 mm (Figure 2). Thus, the length of the object in Figure 2 can be stated as $10{,}5 \pm 0{,}1$ mm.

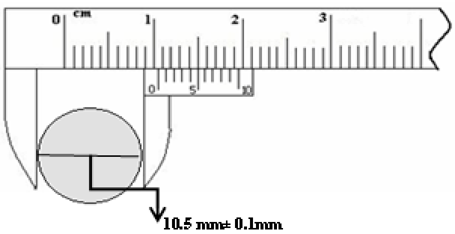


Figure 2. Demonstration of the uncertainty of the vernier caliper.

The micrometer caliper has a linear scale engraved on its sleeve and a circular scale engraved on what is properly called the thimble. Measurements made with a micrometer caliper can be estimated to hundreds of a millimeter. Therefore, the width of the object in Figure 3 can be stated as $13{,}77 \pm 0{,}01$ mm.

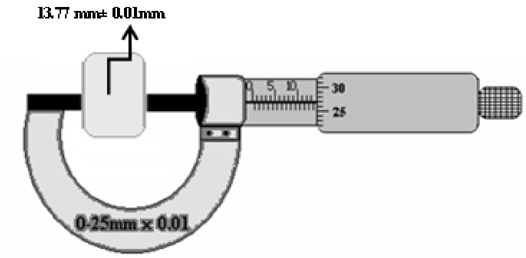


Figure 3. Demonstration of the uncertainty of the micrometer caliper.

Significant Figures

The number of significant figures used in stating a measured value indicates the precision. The number of significant figures in a number is defined as follows:

- The leftmost nonzero digit is the most significant digit.
- If there is no decimal point, the rightmost nonzero digit is the least significant digit.
- If there is a decimal point, the rightmost digit is the least significant digit, even if it is a zero.
- The number of significant figures is the number of digits from the least significant digit to the most significant digit, inclusive.

Examples:

Table 1

Digit	Scientific demonstration	Significant figures
6,23		3
9,1		2
0,00246	$2{,}46\text{x}10^{-3}$	3
0,00000001	1×10^{-8}	1
0,000000010	$1{,}0\times10^{-8}$	2

Arithmetic with Significant Figures

SUMS AND DIFFERENCES: The least significant digit of the result is in the same column relative to the decimal point as the least significant digit of the number entering into the sum or difference which has its least significant digit farthest to the left.

a=5,25 cm and b=2,1 cm

a+b=7,35 cm
~7,4 cm

PRODUCTS AND QUOTIENTS: The number of significant figures in a product or quotient is the same as the number of significant figures in the factor with the fewest significant figures.

a=16,2 cm and b=4,4 cm (2 significant figures)

axb=71,28 cm^2
=71 cm^2 (2 significant figures)

UNITS AND DIMENSIONS

By international agreement a small number of physical quantities such as length, time, mass etc. are chosen and assigned standards. These quantities are called base quantities and their units are base units. All other physical quantities are expressed in terms of these base quantities. The units of these dependent quantities are called derived units.

The units by which we now measure physical quantities is called the S.I. (System International) established in 1960. Within this system, the most commonly used set of units in physics are M.K.S. (Metres, Kilograms, Seconds) system:

Table 2

Physical Quantity				***CGS Unit System***		***MKS Unit System***	
		Dimension		***Unit***		***Unit***	
	Name	**Dimension Symbol**	**Unit Symbol**	**Name**	**Unit Symbol**	**Name**	**Unit Symbol**
Base Quantities	**Length**	[L]	l	centimeter	cm	meter	m
	Mass	[M]	m	gram	g	kilogram	kg
	Time	[T]	t	second	s	second	s
Derived Quantities	**Area**	$[L^2]$	S, A	square centimeter	cm^2	square meter	m^2
	Volume	$[L^3]$	V	cubic centimeter	cm^3	cubic meter	m^3
	Velocity	[L]/[T]	υ	centimeter/ second	cm/s	meter/second	m/s
	Acceleration	$[L]/[T^2]$	a	centimeter/ square second	cm/s^2	meter/ square second	m/s^2
	Force	$[M]x[L]/[T^2]$	F	Dyne	$dyn=g.cm/s^2$	Newton	$N=kg.m/s^2$
	Energy	$[M]x[L^2]/[T^2]$	E	Erg	$Erg= g.cm^2/s^2$	Joule	$J= kg.m^2/s^2$

Powers of Ten

In addition to the basic SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes milli- and nano- denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 3.

Table 3

Prefix	Abbreviation	Power	Prefix	Abbreviation	Power
Deka	dek	10^{1}	**deci**	d	10^{-1}
Hecto	h	10^{2}	**centi**	c	10^{-2}
Kilo	k	10^{3}	**milli**	m	10^{-3}
Mega	M	10^{6}	**micro**	μ	10^{-6}
Giga	G	10^{9}	**nano**	n	10^{-9}
Tera	T	10^{12}	**pico**	p	10^{-12}
Peta	P	10^{15}	**femto**	f	10^{-15}
Exa	E	10^{18}	**atto**	a	10^{-18}

2. Experiment

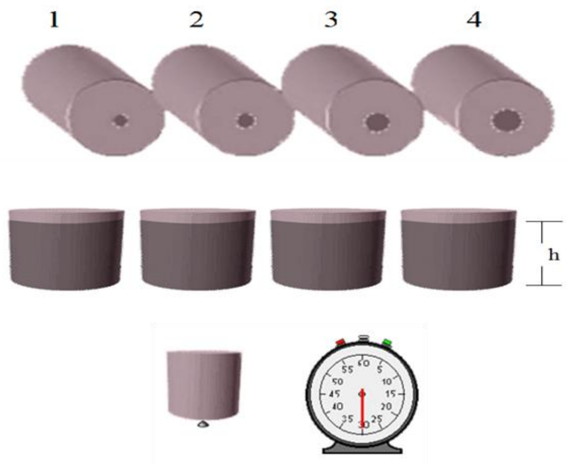


Figure 4

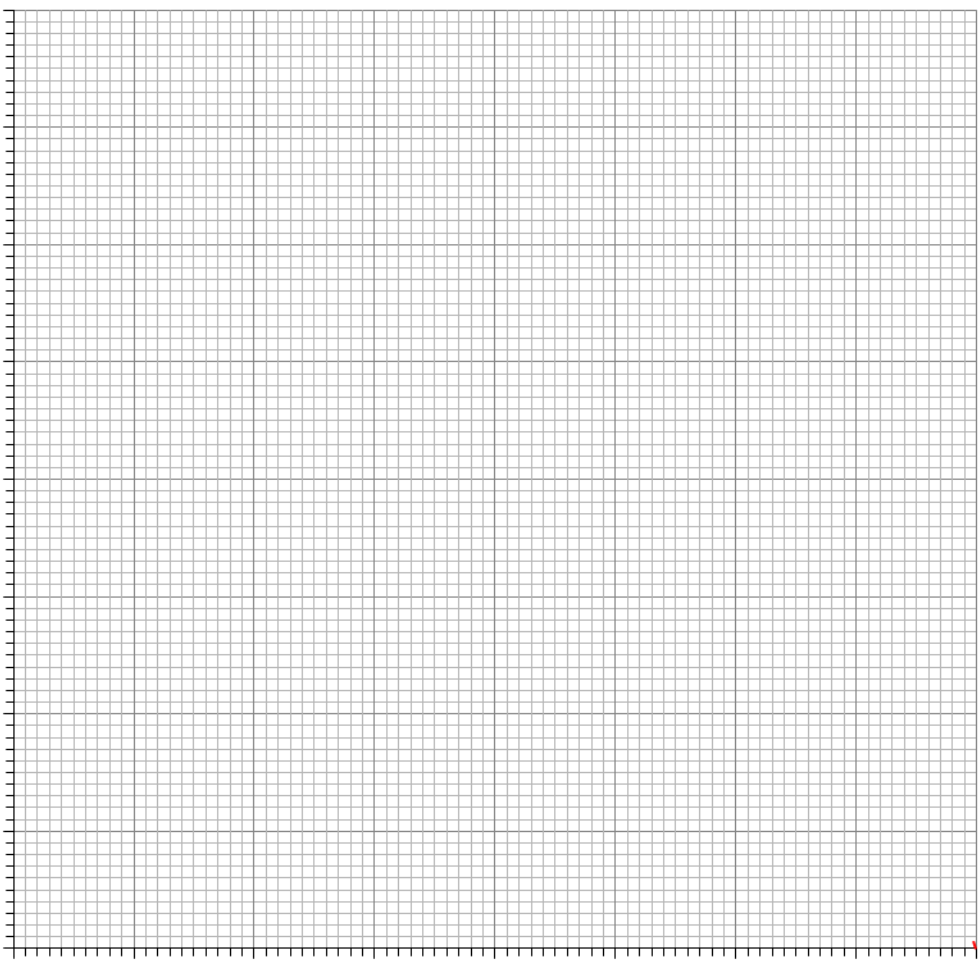
In Table 4, the results of an experiment are presented. The experiment is designed to investigate the pour out time of water through a hole in the bottom of containers. As you would expect, this time depends on the size of the hole and the amount of water in the container. To find the dependence of the pour out time with respect to the hole sizes of containers, we used four holes in different diameters. Then, to find the dependence of the pour out time with respect to the amount of water, containers were filled with water in different heights.

Table 4

Hole diameter d (cm)	Times to empty t (s)	t_{ort}(s)
1.5	72.8	
	73.2	
	73.1	
2.0	41.7	
	41.3	
	41.4	
3.0	18.6	
	18.3	
	18.2	
5.0	6.6	
	7.0	
	6.9	

Analysis

All the information we will use is in Table 4, but a graphical presentation will enable us to make predictions and will greatly facilitate the discovery of mathematical relationships. First, plot the time versus the diameter of the opening for a constant height, for example 30 cm. It is customary to mark the independent variable (in this case, the diameter *d*) on the horizontal axis and the dependent variable (here the time *t*) on the vertical axis. To get maximum accuracy on your plot, you will wish the curve to extend across the whole sheet of paper. Choose your scales on the two axes accordingly, without making them awkward to read.



Graphic 1

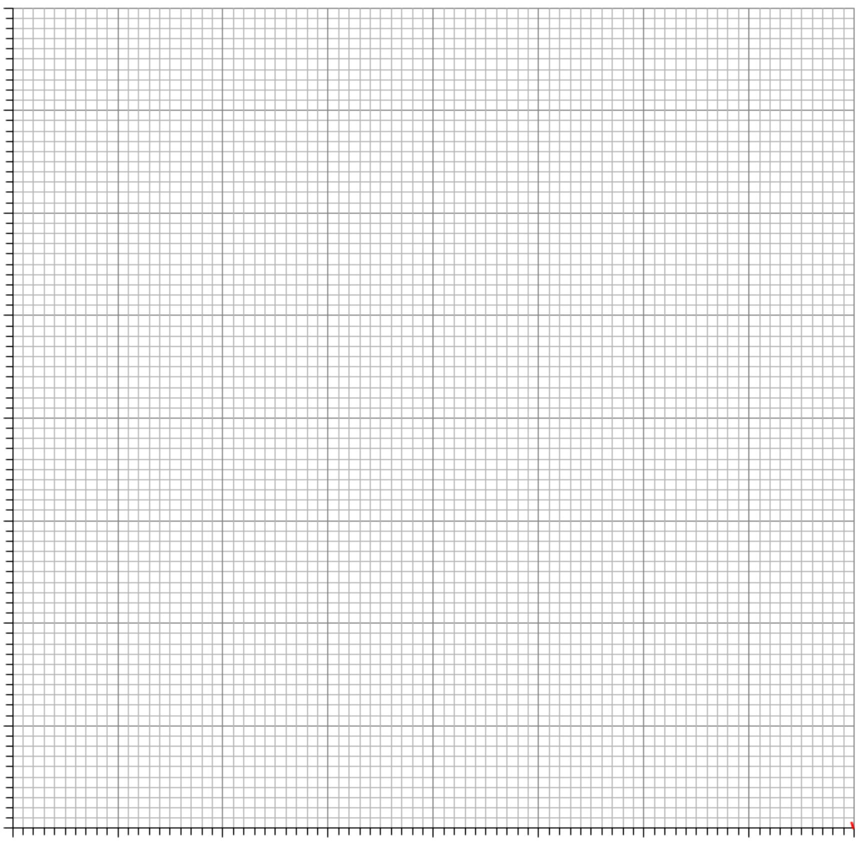
Connect the points by a smooth curve. Is there just one way of doing this? From your curve, how accurately can you predict the time it would take to empty the same container if the diameter of the opening was 4 cm? and 8 cm?

d=4 cm ; t =........ d=8 cm ; t =........

Although you can use the curve to interpolate between your measurements and roughly extrapolate beyond them, you have not yet found an algebraic expression for the relationship between *t* and *d*. From your graph you can see that *t* decreases rather rapidly with *d*; this suggests some inverse relationship. Furthermore, you may argue that the time of flow should be simply related to the area of the opening, since the larger the area of the opening, the more water will flow through it in the same time. This suggests trying a plot of *t* versus $1/d^2$.

Table 5

		n=1	n=2	n=3
t (s)	d (cm)	1/d (cm^{-1})	$1/d^2$ (cm^{-2})	$1/d^3$ (cm^{-3})
73.0	1.5	0.67	0.44	0.30
41.5	2.0	0.50	0.25	0.13
18.3	3.0	0.33	0.11	0.037
6.8	5.0	0.20	0.040	0.0080



Graphic 2

Experimental Errors

All measured quantities contain inaccuracies. These inaccuracies complicate the problem of determining the true value of a quantity. Therefore, the object of experimental work must be to determine the best estimate of the true value of the quantity being measured, together with an indication of the reliability of the measurement.

There are two main sources of experimental errors: Systematic errors and statistical errors.

Systematic errors are associated with the particular instruments or technique used. They can result when an improperly calibrated instrument is used or when some unrealized influence perturbs the system in some definite way, thereby biasing the result of the measurement.

No matter how carefully a measurement is made, it will possess some degree of variability. The errors that result from the lack of precise repeatability of a measurement are called **Statistical errors.** It is often possible to minimize statistical errors by judicious choice of measuring equipment and technique, but they can never be eliminated completely.

Absolute Error

In general, the result of any measurement of physical quantity must include both the value itself and its error. The result is usually quoted in the form

$$\pm \Delta x = x_0 - x$$

where x_0 is the best estimate of what we believe is a true value of the physical quantity and Δx is the estimate of absolute error (uncertainty). Δx indicates the reliability of the measurement, but the quality of the measurement also depends on the value of x_0.

Fractional Error

Fractional error is defined as $\frac{|\Delta x|}{x_o}$.

Fractional error can be also represented in percentile form: $\frac{|\Delta x|}{x_o} \times 100$