EXPERIMENT 5

CONSERVATION OF LINEAR MOMENTUM

Purpose: The purpose of this experiment is verify the law of conservation of linear momentum with the help of the two dimensional collisions.

Equipments: Metal corrugated road, two metal ball (big and small), carbon paper, white paper, ruler, plumb and rope.

1. Theory

I) Momentum: The linear momentum of a particle or an object that can be modeled as a particle of mass *m* moving with a velocity **v** is defined to be the product of the mass and velocity:

$$\vec{P} = m\vec{v} \tag{1}$$

Linear momentum is a vector quantity because it equals the product of a scalar quantity *m* and a vector quantity **v**. Its direction is along v, it has dimensions ML/T, and its SI unit is kg · m/s.

Using Newton's second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle. We start with the Newton's second law and substitute the definition of acceleration:

$$\vec{F}_{ext.} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta\vec{P}}{\Delta t} \tag{2}$$

or

$$\vec{F}_{ext.} = \lim_{\Delta t \to 0}\frac{\Delta\vec{P}}{\Delta t} = \frac{d\vec{P}}{dt} \tag{3}$$

This shows that the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle. The **impulse** of the force **F** acting on a particle equals the change in the momentum of the particle. From the Newton's second Law, Impulse is defined as:

$$\vec{I} = \vec{F}_{ext.} \cdot \Delta t = m\Delta\vec{v} = \Delta\vec{P} \tag{4}$$

When we say that an impulse is given to a particle, we mean that momentum is transferred from an external agent to that particle.

II) Conservation of Momentum: For a system consisting of multiple masses, the total momentum of the system is given by;

$$\vec{P} = \vec{P}_1 + \vec{P}_2 + \cdots = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots = M\vec{v} \tag{5}$$

where *M* is the total mass of the system and $\vec{v}$ is the speed of the center of mass. The total momentum of a system of n particles is equal to the multiplication of the total mass of the system and the speed of the center of mass. So long as the net force on the entire system is zero, the total momentum of the system remains constant (conserved). This is called the **conservation of linear momentum**. Although the momentums of the each particle in the system changes, total momentum stays constant.

$$\vec{P}_{initial} = \vec{P}_{final} \tag{6}$$

III) Collisions: If two bodies collide with each other, they apply a big force to each other in a very short time interval**.** From the Newton's third law, If two objects interact, the force F_{12} exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force F_{21} exerted by object 2 on object 1. The change in the momentum of the object 1:

$$\Delta\vec{P}_1 = \int_{t_1}^{t_2}\vec{F}_{21}\,dt = \vec{F}_{21}\Delta t \tag{7}$$

and the change in object 2:

$$\Delta\vec{P}_2 = \int_{t_1}^{t_2}\vec{F}_{12}\,dt = \vec{F}_{12}\Delta t \tag{8}$$

and if the system is isolated (which means that no external force is acting on the system)

$$\Delta\vec{P}_1 + \Delta\vec{P}_2 = 0 \tag{9}$$

that is, the initial and final mometum of the system is equal to each other. This shows that in a collision, the momentum of the system is conserved.

Collisions are classified as either *elastic* or *inelastic*. Momentum of a system is conserved in all collisions.

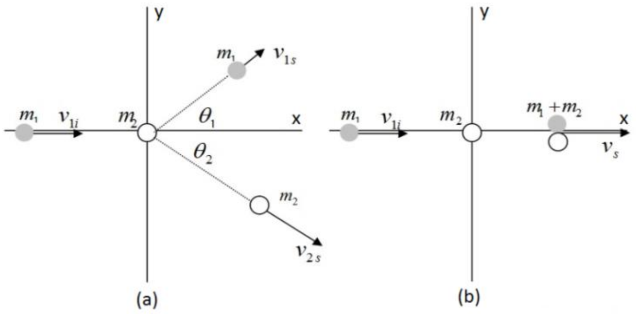

Figure 1. (a) Elastic collision (b) Inelastic collision.

From Figure 1 (a), total momentum of the system in the x-direction before and after the collision:

$$m_1 v_{1i} = m_1 v_{1f}\cos\theta_1 + m_2 v_{2f}\cos\theta_2 \tag{10}$$

and total momentum of the system in the y-direction:

$$m_1 v_f \sin\theta_1 = m_2 v_{2f}\sin\theta_2\,. \tag{11}$$

Kinetic energy of the system is conserved in elastic collisions;

$$\frac{1}{2}m_1 {v_{1i}}^2 = \frac{1}{2}m_1 {v_{1f}}^2 + \frac{1}{2}m_2 {v_{2f}}^2 \tag{12}$$

From Figure 1(b), inelastic collision of two bodies, total momentum of the system in the x-direction before and after the collision is given by,

$$m_1 v_{1i} = (m_1 + m_2)v_s\,. \tag{13}$$

There is no y-component of the momentum and kinetic energy of the system is not conserved in inelastic collisions;

A perfectly elastic collision is defined as one in which there is no loss of kinetic energy in the collision. An inelastic collision is one in which part of the kinetic energy is changed to some other form of energy in the collision. Any macroscopic collision between objects will convert some of the kinetic energy into internal energy and other forms of energy, so no large scale impacts are perfectly elastic.

2. Experiment

1. Tape the paper to the floor and put a carbon paper above it. The plumb bob hangs centered over one edge of the paper and several cm from the end. Mark the position of the plumb. Do not change the position of the paper until the experiment finishes.
2. Write the masses of the balls to the Table-1.

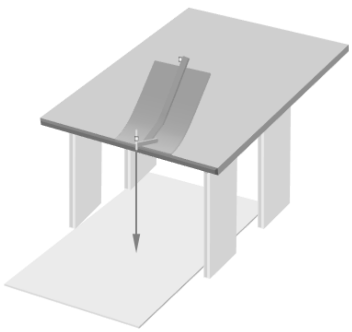

Figure 2. – Conservation of linear momentum apparatus.

3. The position of the support screw can be adjusted by rotating or pulling when the appropriate screws are loosened.
4. Roll the steel ball down the chute 5 times and draw a circle that encloses all the points.

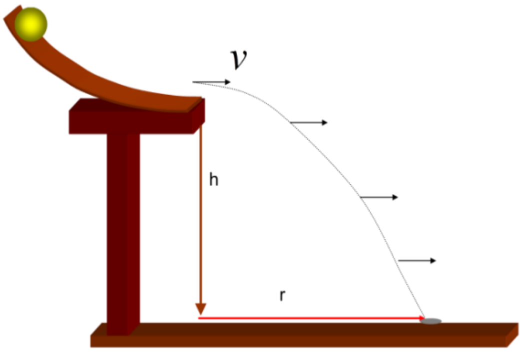

Figure 3. Trajectory of the metal ball.

From Figure 3, the speed of the ball at the end of the corrugated road is;

$$h = \frac{1}{2} g t^2 = \frac{1}{2} g \frac{r^2}{v^2} \tag{14}$$

and given by

$$v = \sqrt{\frac{1}{2} g \frac{r^2}{h}} \tag{15}$$

in here, speed is written as

$$v = \sqrt{\frac{g}{2h}} r = constant\ r \tag{16}$$

and constant is found as

$$constant = \sqrt{\frac{9{,}8}{2 * 0{,}77}} \cong 2{,}52\ s^{-1}$$

The magnitude of the momentum of the ball is calculated with:

$$P = M(kg)\ 2{,}52\ r_i(m) \tag{17}$$

5. Fix the B-arm at an angle (α_1). See Figure 4a,b.
6. Now place the other steel ball on the support screw and roll the projectile ball down the chute to produce a collision. Record the landing positions of the two balls by using carbon paper at the appropriate places. Immediately mark on the paper the points according the collision number and whether it is from projectile or target ball and make sure you mark off unwanted points.

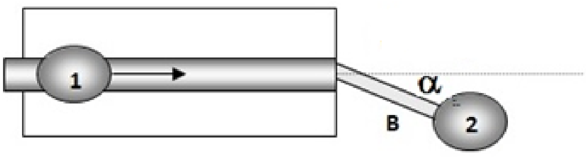

Figure 4a. Top view of the balls.

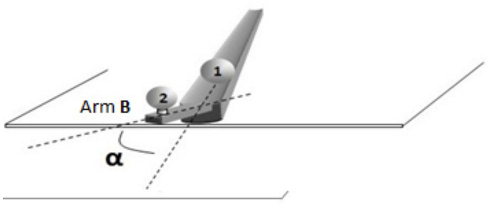

Figure 4b. The position of the small ball at the end of the corrugated road.

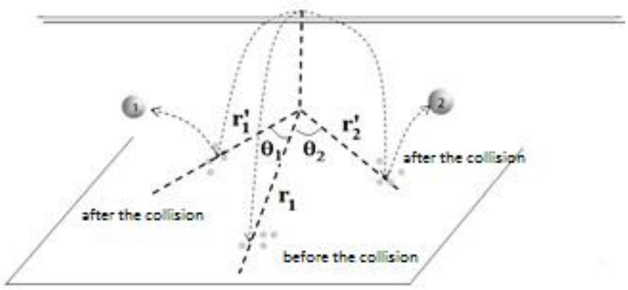

Figure 5. Elastic collision of the balls.

7. Draw a circle that encloses all the points. The centers of the circles are combined with the projection of the plumb (Figure 5) and then r_1, r_1' and r_2' are recorded to the Table-1.

Table 1

$m_{big}(kg)$	$m_{small}(kg)$	$y(m)$	$r_1(m)$	$r'_1(m)$	$r'_2(m)$
0,0112	0,0056	0,77			

8. From Equation 17, calculate the momentum of the balls before $(\vec{P}_1)$ and after $(\vec{P}_1', \vec{P}_2')$ the collision.
9. To show that the momentum is conserved, determine the momentum of the balls in the x and y directions before and after the collision. Use a meter stick to measure the x and y components (**r_{ix}, r_{iy}**). Then, by using the equations below calculate the momentum of the balls in both x and y direcitons.

$P_{ix} = M_i(kg)\ 2{,}52\ r_{ix}(m) \qquad P_{iy} = M_i(kg)\ 2{,}52\ r_{iy}(m)$

Write the results to the Table-2. Calculate the values in Table-3 to show whether the momentum is conserved or not.

Table 2

$P_{1x}(kgm/s)$	$P_{1y}(kgm/s)$	$P'_{1x}(kgm/s)$	$P'_{1y}(kgm/s)$	$P'_{2x}(kgm/s)$	$P'_{2y}(kgm/s)$

Table 3

$P_{1x} = P'_{1x} + P'_{2x}$	$P_{1y} = P'_{1y} + P'_{2y}$