Experiment 7

MOMENT OF INERTIA

Purpose: Measurement of the moments of inertia for rigid objects which rotates around fixed axis.

Equipments: Chronometer, vernier, ruler, disc, ring, plate, masses.

1. Information

Moment of inertia is the physical properties of the objects which rotates. Inertia refers to resistance to change. The moment of inertia is a measure of the resistance of an object to changes in its rotational motion, just as mass is a measure of the tendency of an object to resist changes in its linear motion.

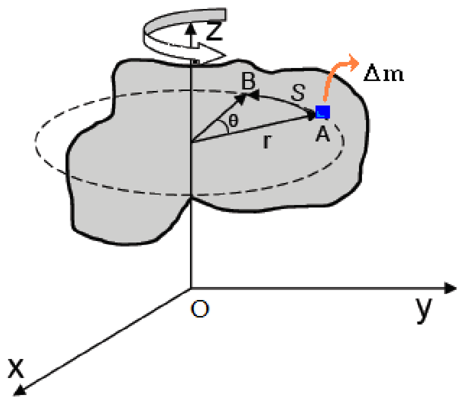

Figure 1. A rigit object which rotates around z-axis.

Rotation Movement: A rigid object which rotate around the z-axis of the inertial coordinate system is given Figure 1. We suppose for this object has a large number of particles of each of the mass Δm. Δm mass's angular position which is far away up to r from the axis of rotation is shown with θ angle. During the rotation in Δt time ıf the angular position change as $\Delta\theta$, particle goes as $\Delta s = r\Delta\theta$. The intensity of linear speed of the particle is given by

$$\upsilon = \frac{dS}{dt} = r\frac{d\theta}{dt} \tag{1}$$

the speed of the change of angle with time is called the angular velocity in (1) and ω express with (2).

$$\omega = \frac{d\theta}{dt} \tag{2}$$

The unit of the ω is rad/s. Then, the equation 1 simplifies to

$$\upsilon = r\omega \tag{3}$$

Angular velocity is same but the linear velocity is different for the all the particles in the rigid object.

Angular Acceleration: The time derivative of ω is called the angular acceleration and is given by

$$\alpha = \frac{d\omega}{dt} \tag{4}$$

The unit of angular acceleration is rad/s^2. Taking time derivative of (3) relation, linear (tangential) acceleration is given by

$$a_t = \frac{d\upsilon}{dt} = r\frac{d\omega}{dt} \tag{5}$$

Accordingly , the relationship with angular acceleration and tangential acceleration (a_t) is given by

$$a_t = r\alpha \tag{6}$$

Rotational Kinetic Energy: Let us consider an object as a collection of particles and assume that it rotates about a fixed *z* axis with an angular speed ω. Figure 1 shows the rotating object and identifies one particle on the object located at a distance r from the rotation axis. Each such particle has kinetic energy determined by its mass and linear speed. If the mass of the *i* th particle is M_i and its linear speed is υ_i , its kinetic energy is

$$K_i = \frac{1}{2}M_i\upsilon_i^2 = \frac{1}{2}M_i r_i^2\omega^2 \tag{7}$$

To proceed further, recall that although every particle in the rigid object has the same angular speed ω, the individual linear speeds depend on the distance r from the axis of rotation according to the expression $\upsilon_i = r\omega$. The total kinetic energy of the rotating rigid object is the sum of the kinetic energies of the individual particles:

$$K = \sum K_i = \frac{1}{2}\sum M_i r_i^2\omega^2 = \frac{1}{2}(\sum M_i r_i^2)\omega^2 \tag{8}$$

where we have factored ω^2 from the sum because it is common to every particle. We simplify this expression by defining the quantity in parentheses as the moment of inertia I:

$$I = \sum M_i r_i^2 \tag{9}$$

From the definition of moment of inertia, we see that it has dimensions of ML^2 (kg·m^2 in SI units). With this notation, Equation 8 becomes

$$E_k = \frac{1}{2}I\omega^2 \tag{10}$$

Although we commonly refer to the quantity $\frac{1}{2}I\omega^2$ as rotational kinetic energy, it is not a new form of energy. It is ordinary kinetic energy because it is derived from a sum over individual kinetic energies of the particles contained in the rigid object.

However, the mathematical form of the kinetic energy given by Equation 10 is convenient when we are dealing with rotational motion, provided we know how to calculate *I*.

It is important that you recognize the analogy between kinetic energy associated with linear motion $\frac{1}{2}mv^2$ and rotational kinetic energy $\frac{1}{2}I\omega^2$. The quantities *I* and ω in rotational motion are analogous to *m* and v in linear motion, respectively. The moment of inertia is a measure of the resistance of an object to changes in its rotational motion, just as mass is a measure of the tendency of an object to resist changes in its linear motion.

Calculation Of Moment Inertia As Using Energy Conservation:

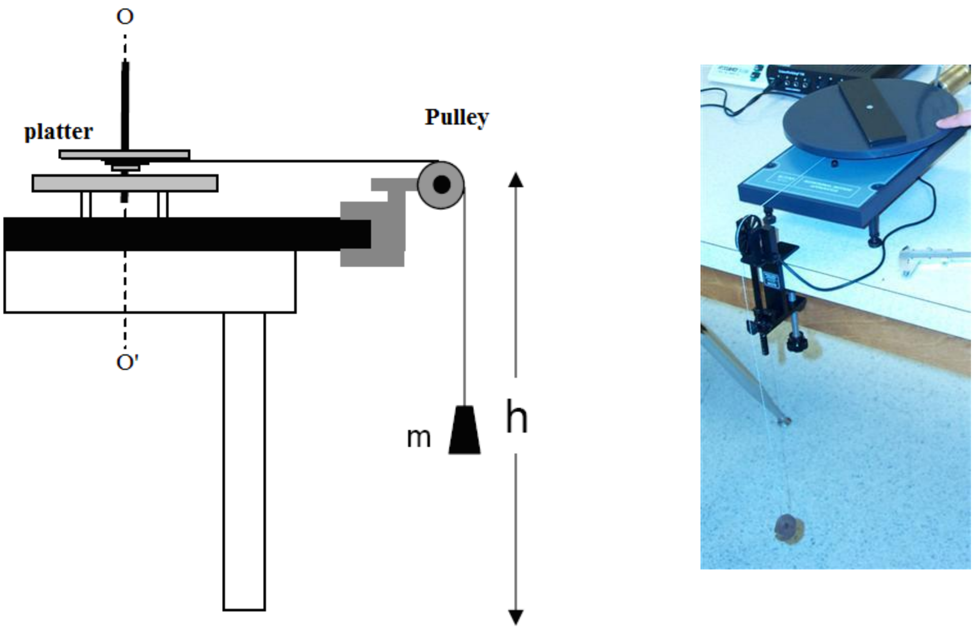

Figure 2. The experimental set up.

Based on the energy conservation,

$$mgh = \frac{1}{2}m\upsilon^2 + \frac{1}{2}I\omega^2 \qquad (12)$$

$$\upsilon = r\omega \tag{13}$$

$$\upsilon = \frac{2h}{t} \tag{14}$$

Using equations (12), (13) and (14) one can find $I_{platter}$ in terms of m, r, g, t, and h.

For the platter:

$$I_{platter} = mr^2\left(\frac{gt^2}{2h} - 1\right) \tag{15}$$

2.Experiment:

Measure the radius of the pulley (r) and write down in Table 1. Height "h" is set to 75 cm (h is the distance from the mass to the ground). Measurement of the landing time for this mass (m=30 g) should be repeated three times. Then calculate the average time. Using equation 15 one can find $I_{platter}$.

Table 1

$\mathbf{r_{pulley}}$ *(m)*	**h** *(m)*	**m** *(kg)*	$\mathbf{t_1}$ *(s)*	$\mathbf{t_2}$ *(s)*	$\mathbf{t_3}$ *(s)*

$t_{average}$ *(s)*	$I_{platter}$ $(kg \cdot m^2)$

Disc:

Disc is placed on to the platter. Height h is set to 75 cm (h is the distance from the mass to the ground). Measurement of the landing time for this mass should be repeated three times. Then calculate the average time. Using equation 15 one can find $I_{platter}+I_{disc}$. Subtract $I_{platter}$ from the total I and find I_{disc}. Calculate the moment of inertia of the disc ($I_{Disc}^{Theoretical}$) from the theoretical formula 16 by measuring the radius of disc and the mass. Then calculate the relative error.

$$I_{Disc^{Theoretical}} = \frac{1}{2} M R^2 \qquad (16)$$

Table 2

r_{pulley} *(m)*	**h** *(m)*	**m** *(kg)*	t_1 *(s)*	t_2 *(s)*	t_3 *(s)*

$t_{average}$ *(s)*	$I_{Platter}+I_{Disc}$ *(kgm²)*	$I_{Disc^{experiment}}$ *(kg·m²)*	M_{Disc} *(kg)*	R_{Disc} *(m)*	$I_{Disc^{Theoretical}}$ (I_{DT}) *(kg·m²)*	R. Error *(%)*

Ring:

Ring is placed on to the platter. Height h is set to 75 cm (h is the distance from the mass to the ground). Measurement of the landing time for this mass (30 g) should be repeated three times. Then calculate the average time. Using equation 15 one can find $I_{platter}+I_{ring}$. Subtract $I_{platter}$ from the total I and find I_{ring}. Calculate the moment of inertia of the ring ($I_{Ring^{Theoretical}}$) from the theoretical formula 17 by measuring the inner and outer radius and the mass. Then calculate the relative error.

$$I_{Ring^{Theoritical}} = \frac{1}{2} M \left(R_{inner}{}^2 + R_{outer}{}^2 \right) \qquad (17)$$

Table 3

r_{pulley} *(m)*	**h** *(m)*	**m** *(kg)*	t_1 *(s)*	t_2 *(s)*	t_3 *(s)*

$t_{average}$ *(s)*	$I_{Platter} + I_{Ring}$ *(kg·m²)*	$I_{Ring^{Experiment}}$ *(kg·m²)*	M_{Ring} *(kg)*	R_{inner} *(m)*	R_{outer} *(m)*	$I_{Ring^{Theoretical}}$ *(kg·m²)*	R. Error *(%)*

Rectangular bar:

Rectangular bar is placed on to the platter. Height h is set to 75 cm (h is the distance from the mass to the ground). Measurement of the landing time for this mass (30 g) should be repeated three times. Then calculate the average time. Using equation 15 one can find $I_{platter}+I_{bar}$. Subtract $I_{platter}$ from the total I and find I_{bar}. Calculate the moment of inertia of the rectangular bar ($I_{Bar^{theoretical}}$) from the theoretical formula 18 by measuring the width (a) , length (b) and the mass. Then calculate the relative error.

$$I_{Bar^{Theoretical}} = \frac{1}{12} M(a^2 + b^2) \qquad (18)$$

Table 4

r_{pulley} *(m)*	**h** *(m)*	**m** *(kg)*	t_1 *(s)*	t_2 *(s)*	t_3 *(s)*

$t_{average}$ *(s)*	$I_{Platter} + I_{bar}$ *(kg·m²)*	$I_{Bar^{Experiment}}$ *(kg·m²)*	M_{bar} *(kg)*	Width (a) *(m)*	Length (b) *(m)*	$I_{Bar^{theoretical}}$ *(kg·m²)*	R.Error *(%)*