# PHYSICS - 1 LABORATORY FOR ENGINEERING STUDENTS FALL SEMESTER



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# Experiment 1 THE ANALYSIS OF AN EXPERIMENT

**Purpose:** To learn how to analyze experimental data and draw graphics, prediction of the results of similar experiments from the mathematical equations which are obtained from analysis, to learn how to make experimental error calculations.

Equipment and materials: Scientific calculator, pencil, eraser, ruler, graph paper.

# 1. Information

The heart of any experiment is making observations and measurements. Accurate measurement requires appropriate tools. While measuring a tabletop, we could use a meter stick to produce a suitable measurement. The meter stick has graduations small enough to attain a measurement within one millimeter. One can make a measurement accurate to within a thousandth of a meter. This is good accuracy if the table is roughly a meter or longer. To use a meter stick to measure the thickness of a pencil would be inappropriate. Assuming a pencil is roughly 5 mm in diameter; one would want a tool that could give measurements accurate to a fraction of a millimeter. The vernier and micrometer calipers were developed to perform such measurements.

All measurements are subject to uncertainty; no matter how precise the instrument that is used or how careful the experiment is done. Therefore it is important to evaluate in some way the magnitude of the uncertainty in a measurement, and if possible, minimize that uncertainty. Consider the following standard metric ruler (Figure 1).

15 mm ±	lmm								
) <sup>mm</sup> 1	2	ż	4	5 2	6	7	8	9	10
inches 1		11	111	цĨ	111	111	Ĩ		цĨ

Figure 1. Demonstration of the uncertainty of the ruler.

The ruler is incremented in units of centimeters (cm). The smallest scale division is a tenth of a centimeter or 1 mm. Therefore, the uncertainty  $\pm$  1 mm. In the example above, the length of the object would be stated as  $15 \pm 1$  mm.

The vernier caliper is an instrument that allows you measure lengths much more accurate than the metric ruler. The smallest increment in the vernier caliper you will be using is 0,1 mm (Figure 2). Thus, the length of the object in Figure 2 can be stated as  $10,5 \pm 0,1$  mm.



Figure 2. Demonstration of the uncertainty of the vernier caliper.

The micrometer caliper has a linear scale engraved on its sleeve and a circular scale engraved on what is properly called the thimble. Measurements made with a micrometer caliper can be estimated to hundreds of a millimeter. Therefore, the width of the object in Figure 3 can be stated as  $13,77 \pm 0,01$  mm.



Figure 3. Demonstration of the uncertainty of the micrometer caliper.

# **Significant Figures**

The number of significant figures used in stating a measured value indicates the precision. The number of significant figures in a number is defined as follows:

- The leftmost nonzero digit is the most significant digit.
- If there is no decimal point, the rightmost nonzero digit is the least significant digit.
- If there is a decimal point, the rightmost digit is the least significant digit, even if it is a zero.
- The number of significant figures is the number of digits from the least significant digit to the most significant digit, inclusive.

Examples:

Table 1						
Digit	Scientific demonstration	Significant figures				
6,23		3				
9,1		2				
0,00246	2,46x10 <sup>-3</sup>	3				
0,0000001	1×10 <sup>-8</sup>	1				
0,000000010	1,0× 10 <sup>-8</sup>	2				

#### **Arithmetic with Significant Figures**

SUMS AND DIFFERENCES: The least significant digit of the result is in the same column relative to the decimal point as the least significant digit of the number entering into the sum or difference which has its least significant digit farthest to the left.

a=5,25 cm and b=2,1 cm a+b=7,35 cm ~7,4 cm PRODUCTS AND QUOTIENTS: The number of significant figures in a product or quotient is the same as the number of significant figures in the factor with the fewest significant figures.

a=16,2 cm and b=4,4 cm (2 significant figures)

axb=71,28 cm<sup>2</sup> =71 cm<sup>2</sup> ( 2 significant figures)

#### UNITS AND DIMENSIONS

By international agreement a small number of physical quantities such as length, time, mass etc. are chosen and assigned standards. These quantities are called base quantities and their units are base units. All other physical quantities are expressed in terms of these base quantities. The units of these dependent quantities are called derived units.

The units by which we now measure physical quantities is called the S.I. (System International) established in 1960. Within this system, the most commonly used set of units in physics are M.K.S. (Metres, Kilograms, Seconds) system:

Physical Quantity				CGS Unit System		MKS Unit System	
		Dimension		Unit		Unit	
	Name	Dimension Symbol	Unit Symbol	Name	Unit Symbol	Name	Unit Symbol
ies	Length	[L]	l	centimetre	cm	metre	m
Quanti	Mass	[M]	m	gram	g	kilogram	kg
Base	Time	[T]	t	second	S	second	S
	Area	[L <sup>2</sup> ]	S, A	square centimetre	cm <sup>2</sup>	square metre	m <sup>2</sup>
5	Volume	[L <sup>3</sup> ]	V	cubic centimetre	cm <sup>3</sup>	cubic metre	m <sup>3</sup>
lantitie	Velocity	[L]/[T]	υ	centimetre/ second	cm/s	metre/second	m/s
rivedQu	Acceleration	[L]/[T <sup>2</sup> ]	а	centimetre/ square second	cm/s <sup>2</sup>	metre/ square second	m/s²
De	Force	[M]x[L]/[T <sup>2</sup> ]	F	Dyne	dyn=g.cm/s <sup>2</sup>	Newton	N=kg.m/s <sup>2</sup>
	Energy	[M]x[L <sup>2</sup> ]/[T <sup>2</sup> ]	Е	Erg	Erg=g.cm <sup>2</sup> /s <sup>2</sup>	Joule	J=kg.m <sup>2</sup> /s <sup>2</sup>

Table 2

#### **Powers of Ten**

In addition to the basic SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes milli- and nano- denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 3.

Prefix	Abbreviation	Power	Prefix	Abbreviation	Power
Deka	dek	101	deci	d	10-1
Hecto	h	10 <sup>2</sup>	centi	с	10-2
Kilo	k	10 <sup>3</sup>	milli	m	10-3
Mega	М	106	micro	μ	10-6
Giga	G	109	nano	n	10-9
Tera	Т	1012	pico	р	10-12
Peta	Р	1015	femto	f	10-15
Exa	E	1018	atto	а	10-18

#### Table 3

#### 2. Experiment



Figure 4

In Table 4, the results of an experiment are presented. The experiment is designed to investigate the pour out time of water through a hole in the bottom of containers. As you would expect, this time depends on the size of the hole and the amount of water in the container. To find the dependence of the pour out time with respect to the hole sizes of containers, we used four holes in different diameters. Then, to find the dependence of the pour out time with respect to the amount of water in the amount of water, containers were filled with water in different heights.

Table 4	1
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Hole diameter	Times to empty	t (a)
<b>d</b> (cm)	<b>t</b> (s)	l <sub>ort</sub> (S)
	72.8	
1.5	73.2	
	73.1	
	41.7	
2.0	41.3	
	41.4	
	18.6	
3.0	18.3	
	18.2	
	6.6	
5.0	7.0	
	6.9	

#### Analysis

All the information we will use is in Table 4, but a graphical presentation will enable us to make predictions and will greatly facilitate the discovery of mathematical relations. First, plot the time versus the diameter of the opening for a constant height, for example, for 30 cm. It is customary to mark the independent variable (in this case, the diameter d) on the horizontal axis and the dependent variable (here the time t) on the vertical axis. To get maximum accuracy on your plot, you will wish the curve to extend across the whole sheet of paper. Choose your scales on the two axes accordingly, without making them awkward to read.



#### Graph 1

Connect the points by a smooth curve. Is there just one way of doing this? From your curve, how accurately can you predict the time it would take to empty the same container if the diameter of the opening was 4 cm? and 8 cm?

d=8 cm; t =.....

Although you can use the curve to interpolate between your measurements and roughly extrapolate beyond them, you have not yet found an algebraic expression for the relationship between t and d. From your graph you can see that t decreases rather rapidly with d; this suggests some inverse relationship. Furthermore, you may argue that the time of flow should be simply related to the area of the opening, since the larger the area of the opening, the more water will flow through it in the same time. This suggests trying a plot of t versus  $1/d^2$ .

		n=1	n=2	n=3	
t (s)	d (cm)	1/d (cm <sup>-1</sup> )	1/d <sup>2</sup> (cm <sup>-2</sup> )	1/d <sup>3</sup> (cm <sup>-3</sup> )	
73.0	1.5	0.67	0.44	0.30	
41.5	2.0	0.50	0.25	0.13	
18.3	3.0	0.33	0.11	0.037	
6.8	5.0	0.20	0.040	0.0080	

Table 5



#### Graph 2

#### **Experimental Errors**

All measured quantities contain inaccuracies. These inaccuracies complicate the problem of determining the true value of a quantity. Therefore, the object of experimental work must be to determine the best estimate of the true value of the quantity being measured, together with an indication of the reliability of the measurement.

There are two main sources of experimental errors: Systematic errors and statistical errors.

**Systematic errors** are associated with the particular instruments or technique used. They can result when an improperly calibrated instrument is used or when some unrealized influence perturbs the system in some definite way, thereby biasing the result of the measurement.

No matter how carefully a measurement is made, it will possess some degree of variability. The errors that result from the lack of precise repeatability of a measurement are called **Statistical errors**. It is often possible to minimize statistical errors by judicious choice of measuring equipment and technique, but they can never be eliminated completely.

#### **Absolute Error**

In general, the result of any measurement of physical quantity must include both the value itself and its error. The result is usually quoted in the form

$$\pm \Delta x = x_0 - x$$

where  $x_0$  is the best estimate of what we believe is a true value of the physical quantity and  $\Delta x$  is the estimate of absolute error (uncertainty).  $\Delta x$  indicates the reliability of the measurement, but the quality of the measurement also depends on the value of  $x_0$ .

#### **Fractional Error**

Fractional error is defined as : 
$$\frac{|\Delta x|}{x_a}$$

Fractional error can be also represented in percentile form:  $\frac{|\Delta x|}{x_a} \times 100$ 

# **Experiment 2**

# **NEWTON'S LAWS OF MOTION**

Purpose: Investigation of Newton's Laws of Motion using air track rail.

**Equipments:** Air track, blower (air source), timer, photogates, vehicles with different masses, masses (10g), rope, pencil, eraser, scientific calculator

# 1. Introduction

# Newton's Laws of Motions:

# I) Newton's First Law of Motion: Law of Inertia

If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration. The resistance to the change in the velocity is called inertia.



The first law of Newton is; If the net force acting on an object is equal to zero (balanced force), the object tends to keep its position. That is, if the object is in rest, it continues in a state of rest, if it is moving, it continues to move without turning or changing its speed.

# II ) Second Law: Law of Motion

When a force acts on an object, the velocity of the object changes, the rate of change of velocity with time is equal to acceleration, so the object gains an acceleration. The second law correlates an kinematic quantity acceleration to a dynamic quantity of force.

Imagine performing an experiment in which you push a block of fixed mass across a frictionless horizontal surface. When you exert some horizontal force on the block, it moves with some acceleration. The acceleration of an object is directly proportional to the force acting on it. The ratio of the force to the acceleration is always constant and it is called, mass.

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \dots = \text{constant}$$
(1)

We can express the second law as: an object under a constant force gains a constant acceleration.

$$\vec{F} = m\vec{a} \tag{2}$$

This is the fundamental equation of dynamics. The equation defines the force.

Force is the quantity that changes the movement of an object. It is an vector and has the same direction with acceleration. The unit of the force in SI unit system is Newton (N). 1 N is the force that gives  $1m/s^2$  acceleration to an object with 1 kg mass. The unit of force in CGS unit system is Dyne.



Newton's second law comprises the first law. If the net force acting on an object is equal to zero then the acceleration becomes zero depending on the fundamental equation and this means the velocity of the object does not change, it becomes constant.

#### III ) Third Law: Action-Reaction Law

Force rises from the interaction of the objects and due to this reason it always exists in pairs. To every action there is always an equal and opposite reaction: or the forces of two bodies on each other are always equal and are directed in opposite directions.



If  $\vec{F}$  and  $-\vec{F}$  forces are the ones that the objects act on each other then,  $\vec{F} = -\vec{F}$ . These forces are called as action-reaction forces.

The ratio of the forces that the objects are applying on each other can be provided with momentum-impulse relation.

Momentum is the product of the mass and the velocity of an object, in other words it is the amount of movement.

$$\vec{p} = m\vec{v}$$

The direction of the momentum is same with the direction of the velocity. The product of the force and its acting time is called **impulse**. If a force,  $\vec{F}_{,}$  is applied on an object for a time dt, then the impulse is  $\vec{F}dt$ . Substitution of equation (3) in equation (2) gives us,

(3)

$$\vec{F} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$
(4)

It can be seen that, the net force that causes acceleration is the the of change of the momentum of a particle with time.

According to equation (4), the impulse applied till the objects are seperated;

$$\overrightarrow{F_1}\Delta t_1 = \overrightarrow{\Delta p_1} \text{ ve } \overrightarrow{F_2}\Delta t_2 = \overrightarrow{\Delta p_2}$$
 (5)

So using equation (2), the ratio of the forces on the objects is,

$$\frac{F_1}{F_2} = \frac{m_1}{m_2} \frac{t_2}{t_1} \frac{l_1}{l_2}$$
(6)

#### 2. Experiment

1. Place the photo-gates as shown in Figure-1 and adjust the air track rail system parallel to the ground with the help of the leg screws.



Figure 1

# I) Application of Newton's 1st Law:

2. Fix the rubber reflector to the left end of the rail and reset the timers. Push the vehicle slowly so it can reach the reflector and return. Read the transition time of the vehicle (t<sub>1</sub>, t<sub>2</sub>) that returned from the reflector and write them down in Table-1.

3. Measure the length of the vehicle ( $\ell$ ) and calculate the velocities of the vehicle passing under the photogates using equation (7), and write down in Table-1 and compare them with to each other. Repeat the experiment with vehicles which have different masses.

l			<i>.</i>
v = -			(7)
t			

Table	1
-------	---

<i>m</i> (kg)	ℓ (m)	<i>t</i> <sub>1</sub> (s)	t <sub>2</sub> (s)	v <sub>1</sub> (m/s)	v <sub>2</sub> (m/s)

4. Place the vehicle on the air track system as the vehicle stays stationary and observe its movement (if the object is stationary, it does not change its position).

#### II ) Application of Newton's 2nd Law:



Figure 2

5. Place the photogates with a distance of s= 0,4 m to each other. Fix a pulley to the end of the rail. The scale is connected to the vehicle by a rope and then hanged from the pulley at the right end of the rail. After the vehicle is located as shown in the figure 2, reset the timers and release the vehicle.

6. Read the transition times of the vehicle passing from two photogates using the timers and write down in Table-2. Add 10 gr masses to the scale and repeat the measurements.

7. Measure the length of the vehicle  $\ell$ .

8. Calculate the velocities of the vehicle passing from the photogates using equation (3), calculate the squares of the values and write down in Table-2.

$$a = \frac{v_2^2 - v_1^2}{2s}$$
(8)

Calculate the acceleration (a) of the mass using equation (8) and write down in Table-2.

$m_{\scriptscriptstyle kefe}$ (kg)	F (N)	<i>t</i> <sub>1</sub> (s)	t <sub>2</sub> (s)	$v_1^2  (m/s)^2$	$v_2^2 ({\rm m/s})^2$	<i>a</i> (m/s²)	$M_{_{system}}$ (kg)	M <sub>vehicle</sub> (kg)
0,03								
0,04								
0,05								
0,06								
	$M_{\it vehicle \ average}$							

Table 2

9. Find the mass of the system  $M_{system}$  using equation (1) which is equal to the sum of the mass of the vehicle and the mass of the scale. Using the equation of  $M_{system} = M_{vehicle} + m_{scale}$  calculate  $M_{vehicle}$ 

$$\frac{\Delta M}{M_{_{vehicle}}^{real}} = \frac{\left| M_{_{vehicle}}^{real} - M_{_{vehicle}}^{ave} \right|}{M_{_{vehicle}}^{real}}$$

#### III ) Application of Newton's 3rd Law:

11. Place the rubber band reflectors on the vehicles and put the vehicles on the rail as the rubber band reflectors become face to face between the photogates as shown in Figure 3 and reset the timers.



Şekil 3

12. Compress the vehicles with equal forces and release the vehicles at the same time. Read the transition times of the vehicles from the timers  $(t_1, t_2)$  and write down in Table-3.

Table 3

<i>m</i> <sub>1</sub> (kg)	<i>m</i> <sub>2</sub> (kg)	<i>t</i> <sub>1</sub> (s)	t <sub>2</sub> (s)	$\frac{m_1}{m_2}$	$\frac{t_1}{t_2}$

13. Find the ratio of the forces acting on two vehicles using equation (6).

 $\frac{F_1}{F_2} = \frac{m_1}{m_2} \frac{t_2}{t_1} \frac{l_1}{l_2} = \dots$ 

14. Repeat the experiment for different masses and fill Table-3.

# **EXPERIMENT 3**

#### CONSERVATION OF LINEAR MOMENTUM

**Purpose:** The purpose of this experiment is verify the law of conservation of linear momentum with the help of the two dimensional collisions.

**Equipments:** Metal corrugated road, two metal ball (big and small), carbon paper, white paper, ruler, plumb and rope.

#### 1. Theory

I) Momentum: The linear momentum of a particle or an object that can be modeled as a particle of mass *m* moving with a velocity **v** is defined to be the product of the mass and velocity:

$$P = mv \tag{1}$$

Linear momentum is a vector quantity because it equals the product of a scalar quantity m and a vector quantity  $\mathbf{v}$ . Its direction is along v, it has dimensions ML/T, and its SI unit is kg  $\cdot$  m/s.

Using Newton's second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle. We start with the Newton's second law and substitute the definition of acceleration:

$$F_{ext.} = ma = m\frac{\Delta v}{\Delta t} = \frac{\Delta P}{\Delta t}$$
(2)

or

$$F_{ext.} = \lim_{\Delta t \to 0} \frac{\Delta P}{\Delta t} = \frac{dP}{dt}$$
(3)

This shows that the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle. The **impulse** of the force **F** acting on a particle equals the change in the momentum of the particle. From the Newton's second Law, Impulse is defined as:

$$I = F_{ext.} \cdot \Delta t = m \Delta v = \Delta P \tag{4}$$

When we say that an impulse is given to a particle, we mean that momentum is transferred from an external agent to that particle.

**II) Conservation of Momentum:** For a system consisting of multiple masses, the total momentum of the system is given by;

$$P = P_1 + P_2 + \dots = m_1 v_1 + m_2 v_2 + \dots = M v$$
(5)

where *M* is the total mass of the system and  $\vec{v}$  is the speed of the center of mass. The total momentum of a system of n particles is equal to the multiplication of the total mass of the system and the speed of the center of mass. So long as the net force on the entire system is zero, the total momentum of the system remains constant (conserved). This is called the **conservation of linear momentum**. Although the momentums of the each particle in the system changes, total momentum stays constant.

$$P_{initial} = P_{final} \tag{6}$$

**III)** Collisions: If two bodies collide, they apply a big force to each other in a very short time interval. From the Newton's third law, If two objects interact, the force  $F_{12}$  exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force  $F_{21}$  exerted by object 2 on object 1. The change in the momentum of the object 1:

$$\Delta P_1 = \frac{t_2}{t_1} F_{21} dt = F_{21} \Delta t \tag{7}$$

and the change in object 2:

$$\Delta P_2 = \frac{t_2}{t_1} F_{12} dt = F_{12} \Delta t \tag{8}$$

and if the system is isolated (which means that no external force is acting on the system)

$$\Delta P_1 + \Delta P_2 = 0 \tag{9}$$

that is, the initial and final mometum of the system is equal to each other. This shows that in a collision, the momentum of the system is conserved.

Collisions are classified as either elastic or inelastic. Momentum of a system is conserved in all collisions.



Figure 1. (a) Elastic collision (b) Inelastic collision.

From Figure 1 (a), total momentum of the system in the x-direction before and after the collision:

$$m_1 v_{1i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2 \tag{10}$$

and total momentum of the system in the y-direction:

$$m_1 v_f \sin \theta_1 = m_2 v_{2f} \sin \theta_2 \,. \tag{11}$$

Kinetic energy of the system is conserved in elastic collisions;

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$
(12)

From Figure 1(b), inelastic collision of two bodies, total momentum of the system in the x-direction before and after the collision is given by,

$$m_1 v_{1i} = (m_1 + m_2) v_s \,. \tag{13}$$

There is no y-component of the momentum and kinetic energy of the system is not conserved in inelastic collisions;

A perfectly elastic collision is defined as one in which there is no loss of kinetic energy in the collision. An inelastic collision is one in which part of the kinetic energy is changed to some other form of energy in the collision. Any macroscopic collisions between objects will convert some of the kinetic energy into internal energy and other forms of energy, so no large scale impacts are perfectly elastic.

#### 2. Experiment

- 1. Tape the paper to the floor and put a carbon paper above it. The plumb bob hangs centered over one edge of the paper and several cm from the end. Mark the position of the plumb. Do not change the position of the paper until the experiment finishes.
- 2. Write the masses of the balls to the Table-1.



Figure 2. – Conservation of linear momentum apparatus.

- 3. The position of the support screw can be adjusted by rotating or pulling when the appropriate screws are loosened.
- 4. Roll the steel ball down the chute 5 times and draw a circle that encloses all the points.



Figure 3. Trajectory of the metal ball.

From Figure 3, the speed of the ball at the end of the corrugated road is;

$$h = \frac{1}{2}gt^2 = \frac{1}{2}g\frac{r^2}{v^2}$$
(14)

and given by

$$v = \frac{1}{2}g\frac{r^2}{h}$$
(15)

in here, speed is written as

$$v = \frac{\overline{g}}{2h}r = constant r$$
(16)

and constant is found as

$$constant = \frac{9,8}{2*0,77} \cong 2,52 \ s^{-1}$$

The magnitude of the momentum of the ball is calculated with:

- 5. Fix the B-arm at an angle  $(\alpha_1)$ . See Figure 4a,b.
- 6. Now place the other steel ball on the support screw and roll the projectile ball down the chute to produce a collision. Record the landing positions of the two balls by using carbon paper at the appropriate places. Immediately mark on the paper the points according the collision number and whether it is from projectile or target ball and make sure you mark off unwanted points.



Figure 4b. The position of the small ball at the end of the corrugated road.



Figure 5. Elastic collision of the balls.

7. Draw a circle that encloses all the points. The centers of the circles are combined with the projection of the plumb (Figure 5) and then  $r_1$ ,  $r'_1$  and  $r'_2$  are recorded to the Table-1.

Table 1

$m_{big}(kg)$	$m_{small}(kg)$	<b>y</b> ( <b>m</b> )	$r_1(m)$	$r'_1(m)$	$r'_2(m)$
0,0112	0,0056	0,77			

- 8. From Equation 17, calculate the momentum of the balls before  $(P_1)$  and after  $(P'_1, P'_2)$  the collision.
- 9. To show that the momentum is conserved, determine the momentum of the balls in the x and y directions before and after the collision. Use a meter stick to measure the x and y components (r<sub>ix</sub>, r<sub>iy</sub>). Then, by using the equations below calculate the momentum of the balls in both x and y directions.

 $P_{ix} = M_i \ kg \ 2,52 \ r_{ix}(m)$   $P_{iy} = M_i \ kg \ 2,52 \ r_{iy}(m)$ 

Write the results to the Table-2. Calculate the values in Table-3 to show whether the momentum is conserved or not.

Table 2

$P_{1x}(kgm/s)$	$P_{1y}(kgm/s)$	$P_{1x}'(kgm/s)$	$P_{1y}^{\prime}(kgm/s)$	$P'_{2x}(kgm/s)$	$P_{2y}^{\prime}(kgm/s)$

Table 3

$\boldsymbol{P}_{1x} = \boldsymbol{P}_{1x}' + \boldsymbol{P}_{2x}'$	$\boldsymbol{P}_{1y} = \boldsymbol{P}_{1y}' + \boldsymbol{P}_{2y}'$

# **Experiment 4**

### **MOMENT OF INERTIA**

Purpose: Measurement of the moments of inertia for rigid objects which rotates around fixed axis.

Equipments: Chronometer, vernier, ruler, disc, ring, plate, masses.

#### 1. Introduction

Moment of inertia is the physical properties of the objects which rotates. Inertia refers to resistance to change. The moment of inertia is a measure of the resistance of an object to changes in its rotational motion, just as mass is a measure of the tendency of an object to resist changes in its linear motion.



Figure 1. A rigit object which rotates around z-axis.

**Rotation Movement:** A rigid object which rotate around the z-axis of the inertial coordinate system is given Figure 1. Suppose that this object has a large number of particles of each of the mass  $\Delta m$ .  $\Delta m$  mass's angular position which is far away up to r from the axis of rotation is shown with  $\theta$  angle. During the rotation in  $\Delta t$  time if the angular position change is  $\Delta \theta$ , particle goes  $\Delta S = r.\Delta \theta$ . The linear speed of the particle is given by

$$\upsilon = \frac{dS}{dt} = r\frac{d\theta}{dt} \tag{1}$$

The (instantaneous) angular velocity  $\omega$ , with which we shall be most concerned, is defined as:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$
(2)

The unit of the  $\omega$  is rad/s. Then, the equation 1 simplifies to

Angular velocity is same but the linear velocity is different for the all the particles in the rigid object.

The (instantaneous) angular acceleration  $\alpha$  is defined as:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$
(4)

The unit of angular acceleration is  $rad/s^2$ . Taking the time derivative of Eq. (3), the linear (tangential) acceleration is given by

$$a_t = \frac{d\upsilon}{dt} = r\frac{d\omega}{dt} \tag{5}$$

Accordingly, the relationship between angular acceleration and tangential acceleration  $(a_t)$  is given by

$$a_t = r\alpha \tag{6}$$

**Rotational Kinetic Energy:** Let us consider an object as a collection of particles and assume that it rotates about a fixed *z*-axis with an angular speed  $\omega$ . Figure 1 shows the rotating object and identifies one particle on the object located at a distance *r* from the rotation axis. Each such particle has a kinetic energy determined by its mass and linear speed. If the mass of the *i* th particle is  $M_i$  and its linear speed is  $v_i$ , its kinetic energy is

$$K_{i} = \frac{1}{2}M_{i}\upsilon_{i}^{2} = \frac{1}{2}M_{i}r_{i}^{2}\omega^{2}$$
(7)

To proceed further, recall that although every particle in the rigid object has the same angular speed  $\omega$ , the individual linear speeds depend on the distance *r* from the axis of rotation according to the expression  $\upsilon_i = r\omega$ . The total kinetic energy of the rotating rigid object is the sum of the kinetic energies of the individual particles:

$$K = \sum K_i = \frac{1}{2} \sum M_i r_i^2 \omega^2 = \frac{1}{2} (\sum M_i r_i^2) \omega^2$$
(8)

where we have factored  $\omega^2$  from the sum because it is common to every particle. We simplify this expression by defining the quantity in parentheses as the moment of inertia I:

$$I = \sum M_i r_i^2 \tag{9}$$

From the definition of moment of inertia, we see that it has dimensions of  $ML^2$  (kg·m<sup>2</sup> in SI units). With this notation, Equation 8 becomes:

$$E_k = \frac{1}{2}I\omega^2 \tag{10}$$

Although we commonly refer to the quantity  $\frac{1}{2}I\omega^2$  as rotational kinetic energy, it is not a new form of energy. It is ordinary kinetic energy because it is derived from a sum over individual kinetic energies of the particles contained in the rigid object. However, the mathematical form of the kinetic energy given by Equation 10 is convenient when we are dealing with rotational motion, provided we know how to calculate *I*. It is important that you recognize the analogy between kinetic energy associated with linear motion  $\frac{1}{2}mv^2$  and rotational kinetic energy  $\frac{1}{2}I\omega^2$ . The quantities *I* and  $\omega$  in rotational motion are analogous to *m* and *v* in linear motion, respectively. The moment of inertia is a measure of the resistance of an object to changes in its rotational motion, just as mass is a measure of the tendency of an object to resist changes in its linear motion.



#### Calculation of moment inertia using energy conservation:

Figure 2. The experimental set up.

Based on the energy conservation,

$$mgh = \frac{1}{2}m\upsilon^2 + \frac{1}{2}I\omega^2 \tag{12}$$

$$v = r\omega$$

$$\upsilon = \frac{2h}{t} \tag{14}$$

(13)

Using equations (12), (13) and (14) one can find  $I_{platter}$  in terms of m, r, g, t, and h.

For the platter:

$$I_{platter} = mr^2 \left(\frac{gt^2}{2h} - 1\right) \tag{15}$$

#### 2.Experiment:

Measure the radius of the pulley (r) and write down in Table 1. Height "h" is set to 75 cm (h is the distance from the mass to the ground). Measurement of the landing time for this mass (m=30 g) should be repeated three times. Then, calculate the average time. Using equation 15, calculate the moment of inertia of platter ( $I_{platter}$ ).

Table 1

r <sub>pulley</sub>	h	m	$t_1$	$t_2$	t <sub>3</sub>
<i>(m)</i>	( <i>m</i> )	( <i>kg</i> )	( <i>s</i> )	( <i>s</i> )	( <i>s</i> )

t <sub>average</sub>	I <sub>platter</sub>
(s)	$(kg \cdot m^2)$

### Disc:

Disc is placed on to the platter. Height "h" is set to 75 cm (h is the distance from the mass to the ground). Measurement of the landing time for this mass should be repeated three times. Then, calculate the average time. Using equation 15, calculate the total moment of inertia for the platter and the disc ( $I_{platter}$ +I <sub>disc</sub>.). Subtract I <sub>platter</sub> from the total I and find I <sub>disc</sub>. Calculate the moment of inertia of the disc ( $I_{Disc}$  <sup>Theoretical</sup>) from the theoretical formula 16 by measuring the radius of disc and the mass. Then, calculate the relative error.

$$I_{Disc}^{Theoretical} = \frac{1}{2}MR^2 \tag{16}$$

Table	2
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<b>r</b> <sub>pulley</sub>	h	m	$t_1$	$t_2$	t <sub>3</sub>
<i>(m)</i>	<i>(m)</i>	(kg)	(s)	(s)	( <i>s</i> )

t <sub>average</sub>	$I_{Platter} + I_{Disc}$	$I_{\it Disc}^{\it Experimental}$	M <sub>Disc</sub>	R <sub>Disc</sub>	$I_{\scriptscriptstyle Disc}^{\scriptscriptstyle Theoretical}\left(\mathrm{I_{DT}} ight)$	R. Error
(s)	$(kgm^2)$	$(kg \cdot m^2)$	(kg)	( <i>m</i> )	$(kg \cdot m^2)$	(%)

# **Ring:**

Ring is placed on to the platter. Height "h" is set to 75 cm (h is the distance from the mass to the ground). Measurement of the landing time for this mass (30 g) should be repeated three times. Then, calculate the average time. Using equation 15 one can find  $I_{platter}+I_{ring}$ . Subtract I <sub>platter</sub> from the total I and find I <sub>ring</sub>. Calculate the moment of inertia of the ring ( $I_{Ring^{Theoretical}}$ ) from the theoretical formula 17 by measuring the inner and outer radius and the mass. Then calculate the relative error.

$$I_{Ring}^{Theoretical} = \frac{1}{2} M \left( R_{inner}^{2} + R_{outer}^{2} \right)$$
(17)

Table 3

r <sub>pulley</sub>	h	m	$t_1$	$t_2$	t <sub>3</sub>
<i>(m)</i>	<i>(m)</i>	(kg)	<i>(s)</i>	<i>(s)</i>	<i>(s)</i>

t <sub>average</sub>	$I_{Platter} + I_{Ring}$	$I_{_{Ring}}^{Experiment}$	M <sub>Ring</sub>	R <sub>inner</sub>	R <sub>outer</sub>	$I_{\it Ring}^{\it Theoretical}$	R. Error
<i>(s)</i>	$(kg \cdot m^2)$	$(kg \cdot m^2)$	(kg)	( <i>m</i> )	( <i>m</i> )	$(kg \cdot m^2)$	(%)

# **Rectangular bar**:

Rectangular bar is placed on to the platter. Height "h" is set to 75 cm (h is the distance from the mass to the ground). Measurement of the landing time for this mass (30 g) should be repeated three times. Then, calculate the average time. Using equation 15 one can find  $I_{platter}$ +I bar. Subtract I platter from the total I and find I bar. Calculate the moment of inertia of the rectangular bar ( $I_{Bar}$ -heoretical) from the theoretical formula 18 by measuring the width (a), length (b) and the mass. Then calculate the relative error.

$$I_{Bar}^{Theoretical} = \frac{1}{12} M (a^2 + b^2)$$
(18)

Table	4
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r <sub>pulley</sub>	h	m	t <sub>1</sub>	$t_2$	t <sub>3</sub>
<i>(m)</i>	<i>(m)</i>	(kg)	( <i>s</i> )	(s)	(s)

t <sub>average</sub>	$I_{Platter} + I_{bar}$	$I_{\scriptscriptstyle Bar}^{\scriptscriptstyle Experimental}$	M <sub>bar</sub>	Width	Length	$I_{\scriptscriptstyle Bar}^{\scriptscriptstyle Theoretical}$	R.Error
<i>(s)</i>	$(kg \cdot m^2)$	$(kg \cdot m^2)$	(kg)	(a)	(b)	$(kg \cdot m^2)$	(%)
	(118 111 )	(118 111 )		<i>(m)</i>	<i>(m)</i>		

# **Experiment 5**

#### SIMPLE PENDULUM AND SPRING PENDULUM

# Aims:

- i) Determination of a spring constant and observation of the period of an object in simple harmonic motion.
- ii) Determination of gravitional acceleration by simple pendulum.

**Equipments:** Spring, strings with different lenghts, pendulum bobs (spheres) and masses, ruler, stopwatch, graphic paper, scientific calculator.

# 1. Introduction

**Simple Harmonic Motion:** A motion which is repeated with certain time intervals is called periodic motion, when an object has a periodic motion around a fixed spot, it is called oscillation.

Harmonic motion is a periodic motion in the form of sinus or cosinus function. When the force acting on an object having harmonic motion is balanced, its position is called equilibrium position and the distance from the equilibrium position in any time is called displacement. If the restoring force acting on the particle to equilibrium position is proportional to displacement, the motion is called simple harmonic motion.

If an object is pulled away and then released, it makes simple harmonic motion. The direction of the restoring force and the displacement are opposite, so that,



Figure- 1. The object in simple harmonic motion.

 $F = -kx \tag{1}$ 

where k is the constant. On the other hand, in the terms on Newton's second law, this restoring force is,

$$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$$
(2)

thus,

$$-kx = m\frac{dv}{dt} \quad \text{or} \quad m\frac{d^2y}{dt^2} + kx = 0$$
(3)

If,  $\omega^2 = k/m$  ( $\omega$ ; angular frequency ) the last equation becomes,

$$\frac{d^2 y}{dt^2} + \omega^2 x = 0 \tag{4}$$

Equation (4) is called harmonic oscillator equation and its solution is,

$$y = Asin \ \omega t + \delta \tag{5}$$

where A is the amplitude and  $\delta$  is the initial phase.

with the help of equation (5) we gain;

dt

$$v = \frac{d x}{dt} = \omega A \cos(\omega t + \delta)$$

$$a = \frac{d v}{dt} = -\omega^2 A \sin(\omega t + \delta) = -\omega^2 y$$
(6)
(7)

The angular frequency can be written as,  $\omega = \frac{2\pi}{T}$ , then the period of the simple harmonic motion is,

$$T = 2\pi \sqrt{\frac{m}{k}}$$
(8)

**Gravitional Acceleration:** When an object is released from a certain height it falls with speeding up. As the object does not have an initial velocity, a force is necessary to start the motion. This can be explained with the laws of dynamics as it gains an acceleration. On the other hand, an object is speeding up in a free fall motion, it is obvious that it has an acceleration. This acceleration acting on the objects is called, gravitional acceleration (g), the force acting on the object is called the weight of the object. If m is the mass of the object,

 $G = mg \tag{9}$ 

In other words, G is the force acting on the force by the earth and it is usually called as gravitional force. Depending on the Newton's third law, as the earth act on the object with a force of G, the object acts a force on the earth as a response.

**Simple Pendulum:** The system with a fixed light string carrying a mass is called simple pendulum (Figure-2). If the mass is pulled over from the equilibrium position and then released, it will make periodic oscillations along the vertical axis with the mg gravitional force and under the tension T on the string. As shown in Figure-1, on the (x, y) plane, the component of mg along the x axis is  $mg \sin \theta$ , the component along the y axis is  $mg \cos \theta$ . Thus the tension on the string T is balanced by  $mg \cos \theta$ 



Figure-2

 $mg\sin\theta$  component is the restoring force and can be written as,

$$F = mgsin\theta \tag{10}$$

If the angle  $\theta$  is small (<5°),  $\sin \theta \approx \theta$  so,  $\theta = x/\ell$  . now the restoring force is,

$$F = -mg\theta = -mg\frac{x}{\ell}$$
(11)

Thus, the restoring force is directly proportional to displacement for small displacement values (F  $\alpha$  x). So the simple pendulum makes simple harmonic motion. According to this, the equation below can be written,

$$F = -kx \tag{12}$$

where k is the spring constant. The (-) sign in equation (12) means it is the restoring force. Using Eqs. (11) and (12),

$$-kx = -mg\frac{x}{\ell} \quad \text{veya} \quad k = \frac{mg}{\ell} \tag{13}$$

Using Newton's second law given by,  $F = m \left( \frac{d^2 x}{dt^2} \right)$ , we can write,

$$-kx = m \left(\frac{d^2 x}{dt^2}\right) \tag{14}$$

using  $\omega^2 = \frac{k}{m}$ , equation (13) becomes

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{15}$$

This is the differential equation of simple harmonic motion. The solution of equation (15) is given below,

$$x = Asin(\omega t + \delta) \tag{16}$$

where A is the constant amplitude value and  $\delta$  is the initial phase. The solution as a function of the initial condition can be written as,

$$x = A\cos(\omega t + \delta) \tag{16a}$$

On the other hand, as  $\omega = \frac{2\pi}{T}$ , the period of the motion is,

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{mg/\ell}} = 2\pi \sqrt{\frac{\ell}{g}}$$
(17)

One can understand that for small oscillations, the period of the simple pendulum is not dependent to the mass of the pendulum bob and the amplitude, it just depends on the length of the pendulum and the gravitional acceleration.

"Note that equation (17) is valid if only the angle heta is small enough."

#### 2. Experiment

#### **Determination of Spring Constant**

1. Hang the mass m at the end of the spring and pull it down softly from the equilibrium position and then release the system. Observe the simple harmonic motion of the system around the equilibrium position.

2. Measure the time for 10 periods to determine the period of the simple harmonic motion. Calculate the average period and write the values down in Table-1.



**Figure 4.** Spring pendulum. The spring lengthens  $\Delta y$  when weight m is hanged. When the spring force equals to the weight of the mass, the system is in equilibrium. If the mass is pulled down y = A from the equilibrium position and then released, simple harmonic motion will be observed.

3. Repeat the experiment by adding extra masses on the spring and calculate the average periods for each mass values. Write the values down in Table-1.

m (kg)	10T (s)	$T_{ave}$ (s)	$T^2$ (s <sup>2</sup> )	k (N/m)

Table-1

4. Plot the  $T^2 = f m$  graph using the values from Table-1. This is supposed to be a straight line passing through the origin point.

5. Calculate the spring constant by equation (8) and choosing two m,  $T^2$  pairs. Write down the results in Table-1.

#### **Determination of Gravitional Acceleration**

6. Measure the length of the string from the hanging point to the sphere  $\ell'$ , and the diameter of the sphere by a caliper and calculate the length of the pendulum  $\ell = \ell' + R$ . Repeat this process for the strings with 4 different lengths and write down the values in Table-2.



7. Pull over the pendulum a little (approximately 5°) and let it to oscillate. Measure the time for 10 periods of oscillation by stopwatch and find the period of the pendulum. (Note that 1 period is the time interval passing, as the pendulum reaches its starting point.)

8. Repeat the experiment bu using strings with different lengths (at least 4 times) and write down the values in Table-2.

2R = 10cm R = 5cm  $\ell' = ....$   $g_{theoretical} = 9,8 m/s^2$ 

$\ell = \ell' + R$ (m)	10T (s)	T <sub>ave</sub> (s)	g <sub>ave</sub> (m/s²)	$\frac{ \Delta g }{g_{theoretical}}$

Table-2

9. Plot the  $T^2 = f \ \ell$  graph with the help of Table-2. Calculate the gravitional acceleration g by the  $\ell/T^2$  ratio from the graph and equation (17).

10. Calculate the relative error on determining the gravitional acceleration comparing the theoretical value from the equation below and write down in Table-2.

