RECITATION-2

(Motion in one dimension)

1-An object moves along the x axis according to the equation $x(t)= (t^3 - 2.00t^2)$ m. Determine

a) the average speed between t = 3.00 s and t = 4.00 s,
b) the instantaneous speed at t = 3.00 s and at t = 4.00 s,
c) the average acceleration between t = 3.00 s and t = 4.00 s, and
d) the instantaneous acceleration at t= 3.00 s and t = 4.00 s.

2- After returning from the bend, a train driver with a speed of 97 km/h recognizes a car 61 m away from the train and moving with a constant speed of 48 km/h. The train driver immediately applies the brake. If the train slows with a constant acceleration, what should be the acceleration in order to avoid from the collision of the train and car?

3- A ball is dropped from rest from a height h above the ground. Another ball is thrown vertically upward from the ground at the instant the first ball is released. Determine the speed of the second ball if the two balls are to meet at a height h/2 above the ground.

4- A stone is thrown straight upward from the edge of the top of a building at an initial speed of 10 m/s. The height of the building is 40 m. How much later must a second stone be dropped from the rest at the same initial height so that the two stones hit the ground at the same time?

5- A test rocket is fired vertically upward from a well. A catapul gives it an initial speed of 80.0 m/s at ground level. Its engines then fire and it accelerates upward at 4.00 m/s^2 until it reaches an altitude of 1 000 m. At that point its engines fail and the rocket goes into free fall, with an acceleration of - 9.80 m/s2.

a) How long is the rocket in motion above the ground?
b) What is its maximum altitude?
c) What is its velocity just before it collides with the Earth? (You will need to consider the motion while the engine is operating separate from the free-fall motion.)

1) $x(t) = t^3 - 2t^2 \ (m)$

a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_s - x_i}{t_s - t_i}$ for $t = 3\,s$; $X_3 = 3^3 - 2.3^2 = 9\,m$

$X_4 = 4^3 - 2.4^2 = 32\,m$

$\bar{v} = \dfrac{x_4 - x_3}{4-3} = \dfrac{32-9}{1} = 23\ m/s$

b) $v = \dfrac{dx}{dt} = \dfrac{d}{dt}(t^3 - 2t^2)$

$v = 3t^2 - 4t$

for $t = 3\,s$; $V_3 = 3.3^2 - 4.3 = 15\ m/s$

$V_4 = 3.4^2 - 4.4 = 32\ m/s$

c) $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_s - v_i}{t_s - t_i}$

$\bar{a} = \dfrac{v_4 - v_3}{4-3} = \dfrac{32-15}{1}$

$\bar{a} = 17\ m/s^2$

d) $a = \dfrac{dv}{dt} = \dfrac{d}{dt}(3t^2 - 4t)$

$a = 6t - 4$

for $t = 3\,s$; $a_3 = 6.3 - 4 = 14\ m/s^2$

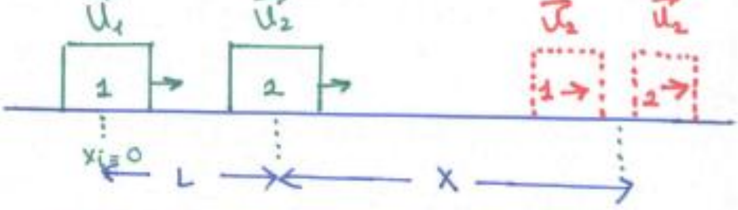
for $t = 4\,s$; $a_4 = 6.4 - 4 = 20\ m/s^2$

2) $L = 61\ m$

$v_1 = 97\ km/h$

$v_2 = 48\ km/h$

$\left(1\ \frac{km}{h} = \frac{10^3}{3600}\ \frac{m}{s}\right)$



* In order to avoid the collision, the maximum speed of the first train must be equal to the speed of the second train, as it arrives the second train.

1^{st} train

$$v_f^2 = v_i^2 + 2a(x_f - x_i) \qquad (v_f)_{max} = v_2$$

$$v_2^2 - v_1^2 = 2a(x+L)$$

$$v_2^2 - v_1^2 = 2ax + 2aL$$

→ the distance traveled by the second train. $\quad x = v_2 . t$

$$v_2^2 - v_1^2 = 2at\,v_2 + 2aL$$

→ the speed of the first train as a function of time $\quad v_2 = v_1 + at$ $\quad \boxed{at = v_2 - v_1}$

$$v_2^2 - v_1^2 = 2(v_2 - v_1)v_2 + 2aL$$

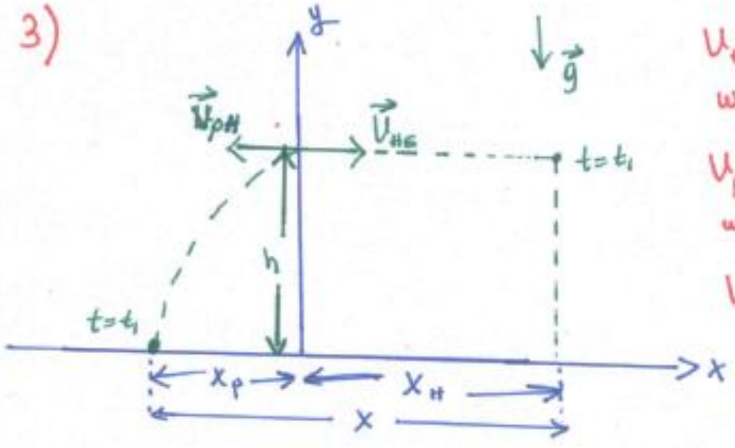
$$\left(\frac{48 \times 10^3}{3600}\right)^2 - \left(\frac{97 \times 10^3}{3600}\right)^2 = 2(48-97).\frac{10^3}{3600} . \frac{48 \times 10^3}{3600} + 2a.61$$

$$a \cong -1.5\ m/s^2$$

$$|a_{min.}| \geq 1.5\ m/s^2$$

in order to avoid the collision...

3)



U_{HG} = Velocity of the helicopter with respect to the ground

U_{PH} = Velocity of the package with respect to the helicopter

U_{PG} = Velocity of the package with respect to the ground

a) $\vec{U}_{PG} = \vec{U}_{PH} + \vec{U}_{HG}$

$\vec{U}_{PG} = (-12 + 6{,}2)\,\hat{i} = -5{,}8\,\hat{i}\ (m/s) = U_{ix} = U_x$

(constant during the motion)

b) $h = \frac{1}{2} g t_1^2$

$t = t_1$ and $y = 0$

$9.5 = \frac{1}{2}(9.8)\,t_1^2$

$t_1 = 1{,}39\ s$ //

$X_P = U_{PG} \cdot t_1$

$X_P = 5.8 \cdot (1{,}39) \cong 8{,}1\ m$

$X_H = U_{HG} \cdot t_1$

$X_H = 6{,}2 \cdot (1{,}39)$

$= 8{,}6\ m$

$X = X_P + X_H = 16{,}7\ m$ //

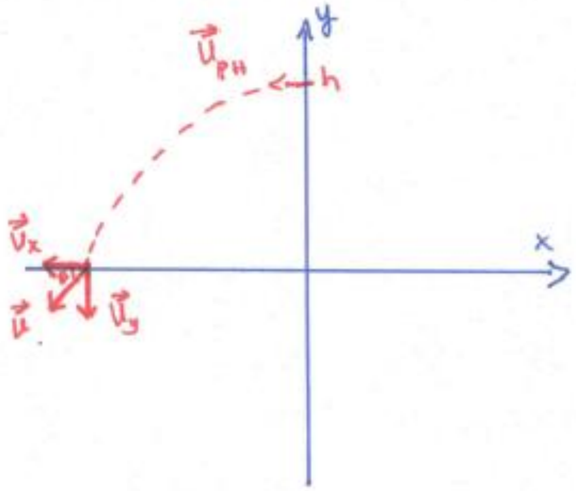
c) $U_x = U_{PG} = -5{,}8\ m/s$

$U_y = -g t_1$

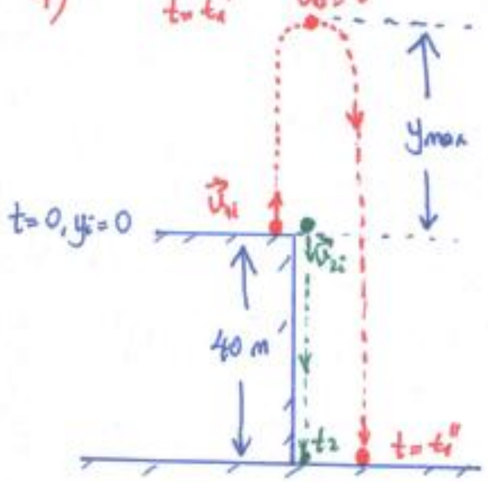
$U_y = -9{,}8 \cdot (1{,}39) \cong -13{,}7\ m/s$

$\theta = \tan^{-1}\left(\frac{U_y}{U_x}\right)$

$\theta = \tan^{-1}\left(\frac{-13{,}7}{-5{,}8}\right) = 67{,}1^\circ$



4)



$* \quad V_f = V_i + at \quad (a = -g)$

$$V_{1f} = V_{1i} - g\,t_1'$$

$$0 = 10 - 9{,}8 \cdot t_1'$$

$$t_1' = 1{,}02 \text{ s}$$

$* \quad y_f - y_i = V_0 t + \frac{1}{2} a t^2$

$$y_{max} - y_i = V_{1i}\,t_1' - \frac{1}{2} g\, t_1'^2$$

$$y_{max} - 0 = 10 \cdot (1{,}02) - \frac{1}{2} 9.8\,(1{,}02)^2$$

$$y_{max} = 5{,}1 \text{ m}$$

* For the first stone, the time taken from the maximum height to the ground;

$$40 + 5{,}1 = \frac{1}{2} \cdot (9{,}8) \cdot t_1''^2 \Rightarrow t_1'' = 3{,}03 \text{ s}$$

* The total time in air, for the first stone;

$$t_1 = t_1' + t_1'' \Rightarrow t_1 = 1{,}02 + 3{,}03 = 4{,}05 \text{ s}$$

* For the second stone, total time to hit the ground;

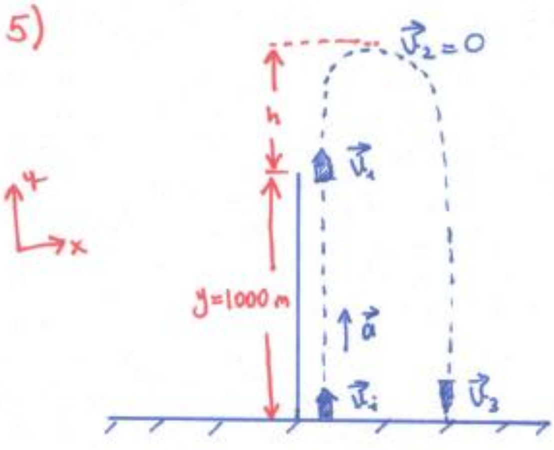
$$40 = \frac{1}{2}\,(9{,}8)\,t_2^2 \Rightarrow t_2 = 2{,}85 \text{ s}$$

$$\Delta t = t_1 - t_2$$

$$\Delta t = 4{,}05 - 2{,}85$$

$$\boxed{\Delta t = 1{,}2 \text{ s}}$$

5)



a) While it is moving in the upward direction;

$v_i = 80\ m/s$
$y_i = 0$
$y_f = 1000\ m$
$a = 4\ m/s^2$

$$y_f - y_i = v_i t_1 + \frac{1}{2} a t_1^2$$

$$1000 - 0 = 80 \cdot t_1 + \frac{1}{2} 4 t_1^2$$

$$1000 = 80 t_1 + 2 t_1^2$$

$$2 t_1^2 + 80 t_1 - 1000 = 0$$

$$\boxed{t_1 = 10 s}$$

* Its speed at 1000 m; v_1

$$v_1 = v_i + a t_1$$

$$v_1 = 80 + 4 \cdot 10$$

$$v_1 = 120\ m/s$$

* When it breaks down;

$$v_2 = v_1 - g t_2$$

$$0 = 120 - 9.8 t_2$$

$$t_2 = 12{,}2\ s$$

* The distance taken after it breaks down;

$$v_2^2 = v_1^2 - 2gh$$

$$0 = (120)^2 - 2.9.8 h$$

$$h = 735\ m$$

b) $$(h + y) = \frac{1}{2} g t_3^2$$

$$1735 = \frac{1}{2} (9.8) t_3^2$$

$$t_3 = 18{,}8 s$$

$$t_{TOTAL} = 10 + 12{,}2 + 18{,}8$$

$$= 41\ s$$ //

c) $$v_3 = v_2 - g t_3$$

$$v_3 = 0 - 9{,}8.(18{,}8)$$

$$v_3 = -184{,}2\ m/s$$

$$\vec{v}_3 = -184{,}2\ \hat{j}\ m/s$$

(Motion in Two dimensions)

1- A ball is thrown from the ground into the air at a certain angle. If at a height of 3 m, the velocity is $v = 4\imath + 3\jmath$ m/s ;

a) Find the velocity of the ball and the angle of the projection of the ball,
b) What is the maximum height reached by the ball?
c) What is the horizontal displacement of the ball?
d) What is the ball's time of flight?

2- The shooter stands on the roof of a 20 m height building. He wants to shoot a target which is on the ground and 50 m away from the base of the building.

a) What should be the initial speed of the ball, if the ball is thrown horizontally.
b) What should be the initial speed of the ball, if the ball the ball is thrown at an angle of 45° to the horizontal.

3- A helicopter is flying in a straight line over a level field at a constant speed of 6.2 m/s and at a constant altitude of 9.5 m. A package is ejected horizontally from the helicopter with an initial velocity of 12 m/s relative to the helicopter, and in a direction to the helicopter's motion.

a) Find the initial speed of the package relative to theground.
b) What is the horizontal distance between the helicopter and thepackage at the instant the package hits the ground?
c) What angle does the velocity vector of the packagemake with the ground at the instant before impact as seen from theground?

4- A train slows down as it rounds the bend and slowing from 108.0 km/h to 72.0 km/h within 150.0 m. The radius of the curve is 200 m. After it moves 100 m in the circular path, find;

a) the tangential acceleration component,
b) the centripetal acceleration component, and
c) the magnitude and direction of the total acceleration

5- Suppose that, on a windy day, an airplane moves with constant velocity of 35.0 m/s towards the south with respect to the air. In this location, there is also a strean of air (wind) with a speed of 10.0 m/s towards the southwest with respect to the ground. By drawing vector diagram, Find the speed and direction of the plane with respect to the ground?

1) $\vec{u} = \vec{u}_x \hat{i} + \vec{u}_y \hat{j}$

$u_x = 4$ m/s

$u_y = 3$ m/s

$\vec{u} = 4\hat{i} + 3\hat{j}$ (m/s)

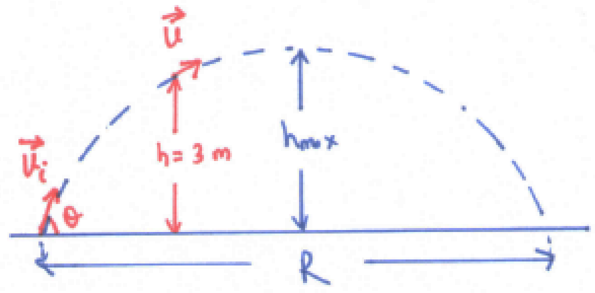
$|\vec{u}| = \sqrt{(4)^2 + (3)^2} = 5$ m/s

$u_f^2 = u_i^2 + 2ax \quad (a = -g)$

$u^2 = u_i^2 - 2gh$

$5^2 = u_i^2 - 2.(9.8).3$

$u_i = 9.2$ m/s



$u_x = u_{ix} = u_i \cos\theta$

$4 = 9{,}2 \cos\theta$

$\cos\theta = 0.43$

$\theta = 64{,}5°$

b) $h_{max} = \dfrac{u_{iy}^2}{2g}$

$h_{max} = \dfrac{(8{,}2)^2}{2.(9{,}8)}$

$h_{max} = 3.4$ m

$u_y = u_{iy}^2 - 2gh$

$3^2 = u_{iy}^2 - 2.(9.8).3$

$u_{iy} = 8{,}2$ m/s

OR

$u_{iy} = u_i \sin 64{,}2$

$u_{iy} = 9{,}2 \sin 64{,}2$

$u_{iy} = 8{,}2$ m/s

c) $R = \dfrac{u_i^2 \sin 2\theta}{g}$

$R = \dfrac{(9{,}2)^2 \sin 128{,}4°}{9.8}$

$R = 6{,}8$ m

d) for h_{max} ; t

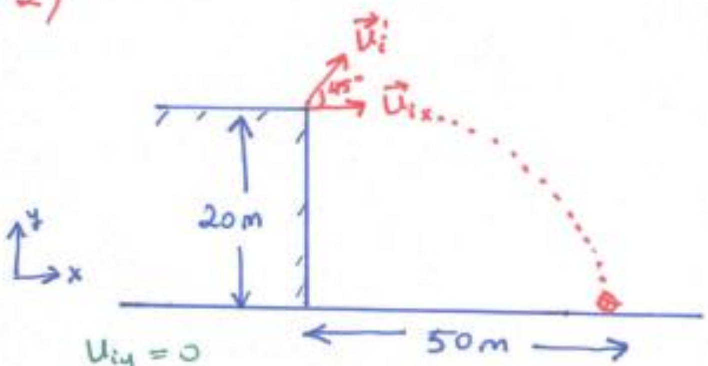
$u_y = 0$, $u_y = u_{iy} - gt$

$t = \dfrac{u_{iy}}{g}$

$t_{flight} = 2.t = \dfrac{2.(8{,}2)}{9{,}8} = 1.7$ s

(total flight time)

2)



$U_{iy} = 0$

$U_{ix} = U_x =$ constant

a) $y_f - y_i = U_{iy} t + \frac{1}{2} a_y t^2$

where $a_y = -g$

$0 - 20 = -\frac{1}{2} \cdot (9.8) \cdot t^2$

$t \cong 2\,s$ //

$X_f - X_i = U_{ix} t + \frac{1}{2} a_x t^2 \quad (a_x = 0)$

$50 = U_{ix} \cdot 2$

$U_{ix} = 25\ m/s$ //

b) $y_f - y_i = U_{iy}' t - \frac{1}{2} g t^2$

$0 - 20 = U_i' \sin 45\, t - \frac{1}{2}\, 9.8\, t^2 \quad (1)$

$X_f - X_i = U_{ix}' t + \frac{1}{2} a_x t^2 \quad (a_x = 0)$

$50 = U_i' \cos 45\, t \quad (2)$ and $t = \dfrac{50}{U_i' \cos 45}$

If we put "t" in Eq. (1)

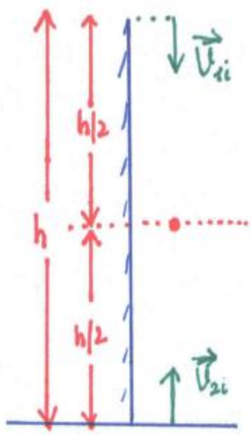
$$0 - 20 = U_i' \sin 45° \left(\frac{50}{U_i' \cos 45}\right) - \frac{9.8}{2}\left(\frac{50}{U_i' \cos 45°}\right)^2$$

$$20 + 50 \cdot \tan 45° = 4.9\ \frac{2500}{U_i'^2 \cos^2 45}$$

$$U_i'^2 \cos^2 45 = \frac{4.9\,(2500)}{70}$$

$$U_i' = 18,7\ m/s \ //$$

3)



$$y_f - y_i = v_i t + \frac{1}{2} a t^2 \quad (a = -g)$$

For the first ball:

$$y_{1f} - y_{1i} = v_{1i} t - \frac{1}{2} g t^2$$

$$\frac{h}{2} - h = 0 - \frac{1}{2} g t^2$$

$$-\frac{h}{2} = -\frac{1}{2} g t^2$$

$$t = \sqrt{\frac{h}{g}} \quad //$$

For the Second ball

$$y_{2f} - y_{2i} = v_{2i} t - \frac{1}{2} g t^2$$

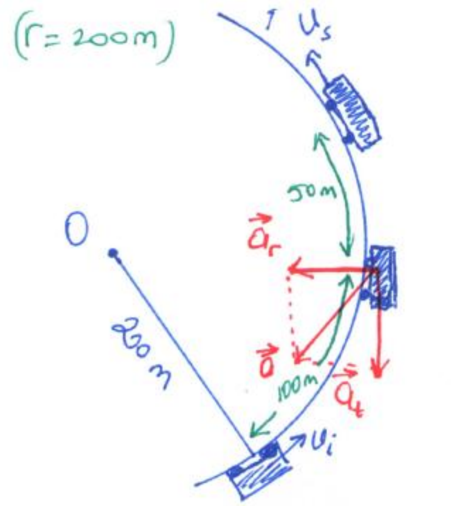
$$\frac{h}{2} - 0 = v_{2i}\sqrt{\frac{h}{g}} - \frac{1}{2} g \left(\sqrt{\frac{h}{g}}\right)^2$$

$$\frac{h}{2} = v_{2i}\sqrt{\frac{h}{g}} - \frac{1}{2} g \frac{h}{g}$$

$$v_{2i} = \sqrt{gh} \quad //$$

4) $v_i = 108 \frac{km}{h} = 108 \cdot \frac{10^3}{3600} = 30\ m/s$

$v_f = 72 \frac{km}{h} = 72 \cdot \frac{10^3}{3600} = 20\ m/s$



$$v_f^2 = v_i^2 + 2a(x_f - x_i) \quad (x_i = 0)$$

$$20^2 = 30^2 + 2a_t \cdot (150)$$

$$400 = 900 + 300 a_t$$

$$a_t = -1.7\ m/s^2$$

(constant during motion)

* Its speed after 100 m; (v')

$$v'^2 = v_i^2 + 2a_t (x_f - x_i)$$

$$v'^2 = 30^2 - 2 \cdot (1.7)(100)$$

$$v'^2 = 560 \Rightarrow v' \cong 23{,}7\ m/s$$

$$a_r = \frac{v'^2}{r}$$

$$a_r = \frac{560}{200} = 2{,}8\ m/s^2$$

$$\vec{a} = \vec{a}_r + \vec{a}_t$$

$$a = \sqrt{a_r^2 + a_t^2}$$

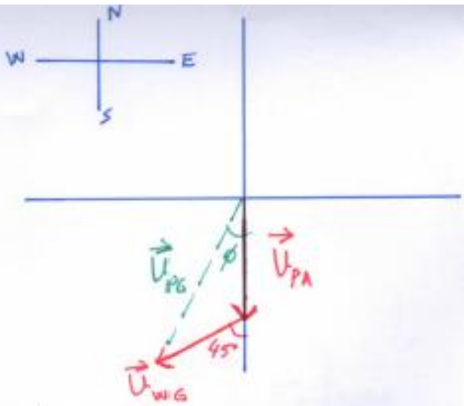
$$a = \sqrt{(2{,}8)^2 + (-1.7)^2}$$

$$a \cong 3{,}3\ m/s^2$$

5) U_{PA} = the velocity of the plane with respect to air

U_{WG} = Velocity of the wind with respect to ground

U_{PG} = Velocity of the plane with respect to ground



$*$ $\vec{U}_{PG} = \vec{U}_{PA} + \vec{U}_{WG}$

$\vec{U}_{PG} = -7{,}07\hat{i} + (-35 - 7{,}07)\hat{j}$

$\vec{U}_{PG} = -7{,}07\hat{i} - 42{,}07\hat{j}$ (m/s)

$U_{PG} = \sqrt{(-7{,}07)^2 + (-42{,}07)^2}$

$U_{PG} = 42.66$ m/s

$\phi = \tan^{-1}\left(\frac{-7{,}07}{-42{,}07}\right)$

$\phi = 9{,}6°$

$*$ $\vec{U}_{PA} = (U_{PA})_x\hat{i} + (U_{PA})_y\hat{j}$

$\vec{U}_{PA} = 0 - 35\hat{j}$

$\vec{U}_{PA} = -35\hat{j}$ (m/s)

$*$ $\vec{U}_{WG} = (U_{WG})_x\hat{i} + (U_{WG})_y\hat{j}$

$\vec{U}_{WG} = -10.\sin 45\hat{i} - 10.\cos 45\hat{j}$

$\vec{U}_{WG} = -7{,}07\hat{i} - 7{,}07\hat{j}$ (m/s)