## Physics-1 Recitation-3

## The Laws of Motion

1) The displacement of a 2 kg particle is given by $x=A t^{3 / 2}$. In here, $A$ is $6.0 \mathrm{~m} / \mathrm{s}^{3 / 2}$. Find the net force acting on the particle. (Note that the force is time dependent).

$$
\begin{aligned}
& x=A t^{3 / 2}, v_{x}=\frac{d x}{d t}=A\left(\frac{3}{2} t^{1 / 2}\right), \\
& a_{x}=\frac{d^{2} x}{d t^{2}}=\frac{d v_{x}}{d t}=\frac{3}{4} A t^{-1 / 2}, \quad \vec{F}_{n e t}=m \vec{a} ; \\
& F=(2 \mathrm{~kg})\left(\frac{3}{4}\right)\left(6 \frac{m}{s^{3 / 2}}\right) t^{-1 / 2}=\left(9 \mathrm{~N} \cdot \mathrm{~s}^{1 / 2}\right)\left(t^{-1 / 2}\right) .
\end{aligned}
$$

2) A particle of mass 2 kg is moving under the action of two forces $\mathbf{F}_{\mathbf{1}}$ and $\mathbf{F}_{\mathbf{2}}$ as shown in the Figure. The force $\mathbf{F}_{1}$ has magnitude 5 N and the force $\mathbf{F}_{\mathbf{2}}$ has magnitude 4 N . At $\mathrm{t}=0$, the particle is at point 0 and its initial velocity is given by $\vec{v}_{\text {initial }}=2 \hat{\mathrm{i}}+\hat{\mathrm{j}}(\mathrm{m} / \mathrm{s})$. Find:
a) Particle's acceleration and its position after 2 seconds in terms of unit vectors.
b) Calculate the angle between the particle's position vector and velocity after 2 seconds.


$$
\begin{aligned}
& \text { a) } \vec{a}=\frac{\sum \vec{F}}{m} \\
& \sum \vec{F}=\sum F_{x} \hat{i}+\sum F_{y} \hat{j} \\
& \Sigma F_{x}=F_{1 x}+F_{2 x}=5 \cdot \cos 37^{\circ}+0=4 \mathrm{~N} \\
& \sum \vec{F}=4 \hat{i}+\hat{j}(N) \\
& \Sigma F_{y}=F_{1 y}+F_{2 y}=-5 \cdot \sin 37^{\circ}+4=1 \mathrm{~N} \\
& \vec{a}=\frac{4 \hat{i}+\hat{j}}{2} \\
& \vec{a}=2 \hat{i}+0,5 \hat{j}\left(\mathrm{~m} / \mathrm{s}^{2}\right) \\
& \vec{r}_{\text {son }}(t)-\vec{r}_{\text {in }}(t)=\vec{v}_{\text {ilk }} t+\frac{1}{2} \vec{a} t^{2} \quad \vec{r}_{\text {ilk }}(t)=0 \\
& \vec{r}(t)=(2 \hat{i}+\hat{j}) t+\frac{1}{2}(2 \hat{i}+0.5 \hat{j}) t^{2} \\
& t=2 \operatorname{sisin} ; \quad \vec{r}(2)=(2 \hat{i}+\hat{j}) \cdot 2+\frac{1}{2}(2 \hat{i}+0,5 \hat{j}) \cdot 2^{2} \\
& \vec{r}(2)=8 \hat{i}+3 \hat{j}(\mathrm{~m})
\end{aligned}
$$

b)

$$
\begin{aligned}
& \vec{v}_{\text {son }}(t)=\vec{v} \text { ilk }(t)+\vec{a} t \\
& \vec{v}(t)=(2 \hat{i}+\hat{j})+\left(2{ }^{i}+0,5 \hat{j}\right) \cdot t
\end{aligned}
$$

$t=2 \operatorname{sisin} ; \quad \vec{v}(2)=(2 \hat{i}+\hat{j})+(2 \hat{i}+0,5 \hat{j}) \cdot 2$

$$
\vec{v}(2)=6 \hat{i}+2 \hat{\jmath}(\mathrm{~m} / \mathrm{s})
$$

$$
\begin{aligned}
\vec{r} \cdot \vec{v} & =r v \cos \theta \\
\cos \theta & =\frac{\vec{r} \cdot \vec{v}}{r v} \\
\cos \theta & =\frac{54}{(8,54) \cdot(6.32)} \\
\cos \theta & =1 \\
\theta & =0^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
& \vec{r}(2)=8 \hat{i}+3 \hat{j}(\mathrm{~m}) \\
& \vec{v}(2)=6 \hat{i}+2 \hat{j}(\mathrm{~m} / \mathrm{s}) \\
& \vec{r}(2) \cdot \vec{v}(2)=(8 \hat{i}+3 \hat{j}) \cdot(6 \hat{i}+2 \hat{j}) \\
& \vec{r}(2) \cdot \vec{v}(2)=54
\end{aligned}
$$

$$
\begin{aligned}
& |\vec{r}(2)|=r(2)=\sqrt{8^{2}+3^{2}}=\sqrt{73}=8,54 \mathrm{~m} \\
& |\vec{v}(2)|=v(2)=\sqrt{6^{2}+2^{2}}=\sqrt{40}=6,32 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

3) Three blocks are in contact with each other on a frictionless, horizontal surface, as in Figure. A horizontal force $\mathbf{F}$ is applied to $m_{1}$. Take $m_{1}=$ $2.00 \mathrm{~kg}, m_{2}=3.00 \mathrm{~kg}, m_{3}=4.00 \mathrm{~kg}$, and $F=18.0 \mathrm{~N}$. Draw a separate freebody diagram for each block and find
(a) the acceleration of the blocks, (b) the resultant force on each block, and (c) the magnitudes of the contact forces between the blocks.

$$
\begin{array}{r}
18 \mathrm{~N}-P=(2 \mathrm{~kg}) a \\
P-Q=(3 \mathrm{~kg}) a \\
Q=(4 \mathrm{~kg}) a
\end{array}
$$


Adding gives $18 \mathrm{~N}=(9 \mathrm{~kg}) a$ so

$$
a=2.00 \mathrm{~m} / \mathrm{s}^{2} \text {. }
$$

(b) $\quad Q=4 \mathrm{~kg}\left(2 \mathrm{~m} / \mathrm{s}^{2}\right)=8.00 \mathrm{~N}$ net force on the 4 kg
$P-8 \mathrm{~N}=3 \mathrm{~kg}\left(2 \mathrm{~m} / \mathrm{s}^{2}\right)=6.00 \mathrm{~N}$ net force on the 3 kg and $P=14 \mathrm{~N}$
$18 \mathrm{~N}-14 \mathrm{~N}=2 \mathrm{~kg}\left(2 \mathrm{~m} / \mathrm{s}^{2}\right)=4.00 \mathrm{~N}$ net force on the 2 kg
(c) From above, $Q=8.00 \mathrm{~N}$ and $P=14.0 \mathrm{~N}$.
4) The assembly in the Right Figure is used to calculate the acceleration of a given system. An observer on the platform can calculate the acceleration of the system by measuring the the angle $(\theta)$ of the ball. In here, $m_{1}$ $=250 \mathrm{~kg}$ and $\mathrm{m}_{2}=1250 \mathrm{~kg}$. Calculate;
a) The acceleration of the system,
b) Derive an equation between the $\theta$ and the acceleration of the system and find $\theta$.
(Suppose that there is no friction between the platform and the table). ( $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$ )


me iain:


$$
\begin{aligned}
& \sum F_{x}=0 \\
& \sum F_{y}=m_{1} g-T_{2}=m_{1} a \\
& \quad T_{2}=m_{1} g-m_{1} a \quad \text { (1) }
\end{aligned}
$$

$$
\sum \vec{F}=m \vec{a}\left\{\begin{array}{l}
\sum F_{x}=m a_{x} \\
\sum F_{y}=m a_{y} \\
\sum F_{z}=m a_{z}
\end{array}\right.
$$

$m_{2}$ iqin:


$$
\begin{aligned}
\sum F_{x}= & T_{2}=m_{2} a \\
\sum F_{y}= & n_{2}-m_{2} g=0 \\
& n_{2}=m_{2} g(3)
\end{aligned}
$$

$m_{3}$ iain:


$$
\begin{aligned}
\sum F_{x}= & T_{1} \sin \theta=m_{3} a \\
\sum F_{y}= & T_{1} \cos \theta-m_{3} g=0 \\
& T_{1} \cos \theta=m_{3} g
\end{aligned}
$$

a)

By using Eqn.(1) and Eqn.(2):

$$
\begin{aligned}
& m_{2} a=m_{1} g-m_{1} a \\
& a\left(m_{1}+m_{2}\right)=m_{1} g \\
& a=\left(\frac{m_{1}}{m_{1}+m_{2}}\right) g
\end{aligned}
$$

$$
\begin{aligned}
m_{1}=250 \mathrm{~kg} & \\
m_{2} & =1250 \mathrm{~kg} \\
g & =3,8 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned} \quad a=\left(\frac{250}{250+1250}\right) 9,8
$$

b) By using Eqn. (4) and Eqn. (5)

$$
\begin{array}{rlr}
\frac{T_{1} \sin \theta}{T_{1} \cos \theta} & =\frac{m_{3} a}{m_{3} g} \\
\tan \theta & =\frac{a}{9} \\
\theta & =\tan ^{-1}\left(\frac{a}{9}\right) \quad a=1,63 \mathrm{~m} / \mathrm{s}^{2} & \theta \\
\theta & =\tan ^{-1}\left(\frac{1,63}{9,8}\right) \\
\theta & =9,4^{\circ}
\end{array}
$$

5) $A, B$ and $C$ are connected by massless strings that passes over frictionless pulleys. The weights of $A$ and $B$ are given as 25 N . The coefficients of kinetic friction between the block A and the ground, and the Block $B$ and incline is 0.35 . When the system is released, block C moves downward with a constant speed.
a) Draw free-body diagrams of objects and find the tension in the string between the Block A and B.

b) Find the weight of Block C.
c) If we cut the string between blocks $A$ and $B$, What will be the acceleration of Block C ? $\left(\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}\right)$.


c) $\Sigma \vec{F}=m \vec{a}\left\{\begin{array}{l}\Sigma F_{x}=m a_{x} \\ \Sigma F_{y}=m a_{y}\end{array}\right.$

For Block B

$$
\begin{aligned}
& \sum F_{X}=T_{2}-m_{B} g \sin 37^{\circ}-f_{k B}=m_{B} a \\
& \sum F_{y}=n_{B}-m_{B} g \cos 37^{\circ}=0 \\
& n_{B}=m_{0} g \cos 37^{\circ} \\
& n_{B}=25 \cdot \cos 37^{\circ} \\
& n_{B}=20 \mathrm{~N}
\end{aligned}
$$

For Block C


$$
\begin{aligned}
& \sum F_{x}=0 \\
& \sum F_{y}=m_{c} g-T_{2}=m_{c} a \\
& \quad T_{2}=m_{c} g-m_{c} a
\end{aligned}
$$

$$
m_{c} g=31 \mathrm{~N}
$$

$$
m_{c}=\frac{31}{9,8}=3,2 \mathrm{~kg}
$$

$$
\operatorname{mog}=25 \mathrm{~N}
$$

By using eqn. (6) and . Eqn. (7)

$$
M_{B}=\frac{25}{9,8} \cong 2,6 \mathrm{~kg}
$$

$$
\begin{aligned}
m_{C} g-m_{C} a & =m_{B} a+m_{B} g \sin 37^{\circ}+\mu_{k} n_{B} \\
31-3,2 a & =2,6 a+25 \cdot \sin 37^{\circ}+0,35.20 \\
a & \cong 1,54 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

6) In the Figure, The coefficient of kinetic friction between the $M_{1}$ and $M_{2}$ and the rough table is 0.5 . (Ignore the masses of the pulleys and the friction on the strings). Find;
a) The tension in the strings,
b) The acceleration of each mass. $\left(m_{1}=2 \mathrm{~kg}, \mathrm{~m}_{2}=8 \mathrm{~kg}, \mathrm{~m}_{3}=4 \mathrm{~kg}, \mathrm{~g}=10 \mathrm{~m} / \mathrm{s}^{2}\right)$.

```
\(f_{k_{1}}=\mu_{k \cdot A_{1}}=\mu_{k} m_{1} g\)
\(f_{k A}=0,5 \cdot 2 \cdot 10\)
\(f_{k_{1}}=10 \mathrm{~N}\)
```

By using Eqns. (1), (2) and (3):

$$
\begin{aligned}
& a_{1}=\frac{T-f_{k 1}}{m_{1}}=\frac{T-10}{2} \\
& a_{2}=\frac{m_{2 g}-2 T}{m_{2}}=\frac{80-2 T}{8} \\
& a_{3}=\frac{T-f_{k 3}}{m_{3}}=\frac{T-20}{4}
\end{aligned}
$$

$$
\text { b) } a_{1}=\frac{24-10}{2}=7 \mathrm{~m} / \mathrm{s}^{2}
$$

$$
a_{2}=\frac{80-2.24}{8}=4 \mathrm{~m} / \mathrm{s}^{2}
$$

$$
a_{3}=\frac{24-20}{4}=1 \mathrm{~m} / \mathrm{s}^{2}
$$

$f_{k_{3}}=\mu_{k} \cdot n_{3}=\mu_{k} m_{3} 9$
$f_{k_{3}}=0,5.4 .10$
$f k_{3}=20 \mathrm{~N}$


The total distance taken by $m 1$ and $m 3$ is equal to $\times 1+x 3$. Then,
The distance taken by mass m 2 is: $\mathrm{x} 2=(\mathrm{x} 1+\mathrm{x} 3) / 2$

$$
\begin{aligned}
x=\frac{1}{2} a t^{2} \text { and } a_{2} & =\frac{a_{1}+a_{3}}{2} \\
2 a_{2} & =a_{1}+a_{3}
\end{aligned}
$$

$2 \cdot\left(\frac{80-2 T}{8}\right)=\frac{T-10}{2}+\frac{T-20}{4}$

$$
T=24 \mathrm{~N}
$$

7) In the Figure, the coefficient of kinetic friction between $\mathrm{M}_{2}$ and the table is 0.2 . When the system is released, in order to avoid the slipping of $M_{3}$ over $M_{2}$, what should be the magnitude of the static friction between the $M_{3}$ and $M_{2}$ ?


8) In Figure, the coefficient of kinetic friction between $M_{1}$ and $M_{2}$ and also between $M_{2}$ and table is 0.2 . $\mathrm{M}_{1}$ is pulled with a 10 N force as shown in Figure:
a) Find the acceleration of the system and

b) The tension in the string.

$$
\left(m_{1}=1 \mathrm{~kg}, \mathrm{~m}_{2}=2 \mathrm{~kg}, \mathrm{~g}=10 \mathrm{~m} / \mathrm{s}^{2}\right)
$$



Using eqns. (1) and (2):
$f \cos 37^{\circ}-f_{k_{1}}-m_{1} a=f k_{i}+f k_{2}+m_{2} a$
$f \cos 37^{\circ}-f_{k_{1}}-f_{k_{1}}-f_{k_{2}}=\left(m_{1}+m_{2}\right) a$

$$
\begin{aligned}
& f_{k 1}=f_{k_{1}}^{\prime}=\mu_{k} n_{1}=0,2.4=0,8 \mathrm{~N} \\
& f_{k 2}=\mu_{k} \cdot n_{2}=0,2 \cdot 24=4,8 \mathrm{~N}
\end{aligned}
$$

$10 \cdot \cos 37^{\circ}-0,8-0,8-4,8=(1+2) a$

$$
a \cong 0,53 \mathrm{~m} / \mathrm{s}^{2}
$$

$$
\text { b) } \begin{aligned}
T & =F \cos 37^{\circ}-f_{k 1}-m_{1} a \\
T & =10 \cdot \cos 37^{\circ}-0,8-2 \cdot 0,53 \\
T & \cong 6,67 \mathrm{~N}
\end{aligned}
$$

## Circular Motion and Other

## Applications of Newton's Laws

1) A puck of mass $m_{A}=35 g$ slides in a circle of radius $r=0.4 \mathrm{~m}$ on a frictionless table while attached to a hanging mass $m_{B}=25 \mathrm{~g}$ by means of a cord that extends through a hole in the table.
a) What speed keeps the mass $B$ at rest?
b) For the situation in part a), Calculate the acceleration of $A$ and write the acceleration


$$
\begin{align*}
& \text { a) For } \mathrm{A} \\
& \sum F_{x}=T=m_{A} \cdot a_{r} \\
& T=m_{A} \frac{v^{2}}{r}  \tag{1}\\
& \sum F_{y}=n_{A}-m_{A} g=0 \\
& n_{A}=m_{A} g \\
& \text { b) } v=\text { sabin }\left(a_{t}=0\right) \\
& \vec{a}=\vec{a}_{r} \\
& a r=\frac{v^{2}}{r} \\
& a_{r}=\frac{(1,67)^{2}}{0.4}=7 \mathrm{~m} / \mathrm{s}^{2} \\
& \vec{a}=7 \hat{r}\left(\mathrm{~m} / \mathrm{s}^{2}\right) \\
& \text { For B } \\
& \Sigma F_{x}=0 \\
& \Sigma F_{y}=m_{B} g-T=0 \\
& T=m_{0} g \text { (2) } \\
& \text { Using Ens. (1) and (2): } \\
& m_{A} \frac{v^{2}}{r}=m_{0} g \\
& v=\sqrt{\frac{25 \cdot 10^{-3} \cdot 9,8 \cdot 0,4}{35 \cdot 10^{-3}}} \\
& v=1,67 \mathrm{~m} / \mathrm{s}
\end{align*}
$$

2) A small coin is placed on a turntable making 3 revolutions in 3.14 s .
(a) What are the speed and acceleration of the coin when it rides without slipping at 5.0 cm from the center of the turntable? (b) What is the magnitude and direction of the friction force if the mass of the coin is 2.0 g ? (c) What is the coefficient of static friction if the coin is observed to slide off when it is more than 10 cm from the center of the turntable?
(a) $v=\frac{2 \pi r}{T}$

$$
=\frac{2 \pi r}{\frac{\pi}{3}}
$$

$$
=6 r
$$

$$
=0.3 \mathrm{~m} / \mathrm{s}
$$



$$
a=\frac{v^{2}}{r}
$$

(b)

$$
\begin{aligned}
& =1.82 \mathrm{~m} / \mathrm{s}^{2} \\
f & =m a
\end{aligned}
$$

$$
=3.64 \times 10^{-3} \mathrm{~N}
$$

(c) At $r=10 \mathrm{~cm}$, the friction is just enough to provide the centripetal force:

$$
\begin{aligned}
\mu N & =\frac{m v^{2}}{r} \\
\mu m g & =\frac{m v^{2}}{r} \\
\mu & =\frac{v^{2}}{g r} \\
& =\frac{\left(\frac{2 \pi r}{T}\right)^{2}}{g r} \\
& =\frac{4 \pi^{2} r}{g T^{2}} \\
& =\frac{4 \pi^{2} r}{g\left(\frac{\pi}{3}\right)^{2}} \\
& =\frac{36 r}{g} \\
& =0.37
\end{aligned}
$$

3) A student of mass 68 kg rides a steadily rotating Ferris wheel (the student sits upright). At the highest point, the magnitude of the normal force on the student from the seat is 556 N .
a) What is the magnitude of the normal force on the student at the lowest point?
b) If the wheel's speed is doubled, what is the magnitude $F_{N}$ at the highest point?
4) 

a) At the Bottom
$F_{\text {net }}=m \frac{v^{2}}{R}$
$n$ alt $-m g=m \frac{v^{2}}{R}$
nat $=m g+m \frac{v^{2}}{R}$ (1)

At the Top
Fret $=m g-n$ Hurst
$F_{\text {net }}=m \frac{v^{2}}{R}$
$m g-n_{i s t}=m \frac{v^{2}}{R}$
nuist $=m g-m \frac{v^{2}}{R}$
$m \frac{v^{2}}{R}=m g-n_{i s t}$
(2)

nüst $=556 \mathrm{~N}$

Using Eqn. (3) and Eqn (1):
$n_{\text {alt }}=m g+m g-n_{i s t}$
$n_{\text {alt }}=2.68 .9 .8-556$
Malt $\cong 777 \mathrm{~N}$
b) $v \rightarrow 2 v$ and $F_{\text {net }} \frac{v^{2}}{R} \quad\left(F \propto v^{2}\right) F_{\text {net }} \rightarrow 4 F_{\text {net }}$

By using Eqn. (2):

$$
\begin{aligned}
& n_{\text {uss }}^{\prime}=m g-4 \frac{v^{2}}{R} \Leftrightarrow \text { from Eqn. (3) } \\
& n_{\text {iss }}^{\prime}=m g-4\left(m g-n_{\text {is }}\right) \\
& n_{\text {üst }}^{\prime}=68.9,8-4(68.9,8-556) \\
& n_{\text {inst }}^{\prime} \cong 225 \mathrm{~N}
\end{aligned}
$$

4) In an old-fashioned amusement park ride, passengers stand inside a 5.0 m diameter hollow steel cylinder with their backs against the wall. The cylinder begins to rotate about a vertical axis and reaches $0.60 \mathrm{rev} / \mathrm{sec}$ constant speed. Then the floor on which the passengers are standing suddenly drops away! If all goes well, the passengers will "stick" to the wall and not slide.
a) Draw the free body diagram for the person inside the cylinder after the floor drops away and,
b) Find the minumum static coefficient of friction in order to keep the person inside the cylinder.
5) 


b) The maximum static friction force equals the person's weight.

The normal force exerted on the person by the cylindirical wall must be equal to the centripetal force.


$$
n=\frac{m v^{2}}{R}
$$

$$
\begin{aligned}
\sum F_{x}=n & =m \cdot a_{r} \\
n & =m \frac{v^{2}}{r}(1)
\end{aligned}
$$

$$
\begin{gathered}
\Sigma f_{y}=m g-f_{s}=0 \\
\mu_{s \cdot n}=m g
\end{gathered}
$$

$$
\begin{equation*}
n=\frac{m g}{\mu_{s}} \tag{2}
\end{equation*}
$$

$$
\begin{array}{lr}
v=w \cdot r=2 \pi f r & \text { using eqns. (1) and (2): } \\
v=2 \pi \cdot 0,60 \cdot 2,5 & m \frac{v^{2}}{r}=\frac{m g}{\mu_{s}} \\
v=9,42 \mathrm{~m} / \mathrm{s} & \mu_{s}=\frac{r g}{v^{2}} \\
\mu_{s}=\frac{2,5 \cdot 9 \cdot 8}{(3,42)^{2}} \\
& \mu_{s} \cong 0,28
\end{array}
$$

5) In Figure, a 4.0 kg ball is connected by means of two massless strings, each of length $L 1.25 \mathrm{~m}$, to a vertical, rotating rod. The strings are tied to the rod with separation $d=2.0 \mathrm{~m}$ and are taut. The tension in the upper string is 80 N.What are the
(a) tension in the lower string and ,
(b) How many revolution per minute does it make?

a)


$$
\begin{aligned}
\Sigma F_{y}= & T_{\text {oust }} \cdot \sin \theta-T_{\text {alt }} \cdot \sin \theta-m g=0 \\
& 80 \cdot \sin 53^{\circ}-T_{\text {alt. }} \cdot \sin 53^{\circ}-4 \cdot 9,8=0 \\
& \text { Talk }_{\cong} 31 \mathrm{~N}
\end{aligned}
$$

b) $\sum F_{x}=T_{\text {inst }} \cos \theta+T_{\text {alt }} \cos \theta=m a_{r}=m \frac{v^{2}}{r}$

$$
(80+31) \cdot \cos 53^{\circ}=4 \frac{v^{2}}{0,75}
$$

$$
v=3,53 \mathrm{~m} / \mathrm{s}
$$

$v=\omega r=2 \pi f r$
$f=\frac{v}{2 \pi r}$
$f=\frac{3,53}{2 \pi 0,75}$
1 side 0,75 devir
1 dak (60s) $f^{\prime}$
$f=0,75 \mathrm{rev} / \mathrm{min}$

$$
\begin{aligned}
& f^{\prime}=0.75 .60 \\
& f^{\prime}=45 \mathrm{rev} / \mathrm{min}
\end{aligned}
$$

