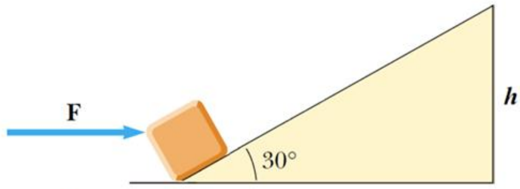
RECITATION-4

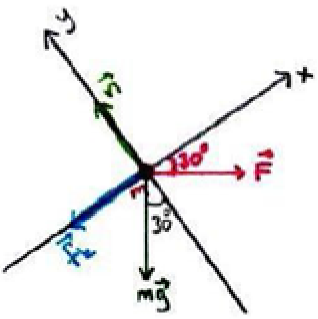
Work, Kinetic Energy,Potential Energy and Conservation of Energy

1) A 200 *N* block is pushed up a frictionless, 30°, 3 *m* inclined plane by a force *F* parallel to the inclined plane. The speed of the block at the bottom of the inclined plane is 0.5 *m/s* and 4 *m/s* at the top. Draw the free body diagram and find;



a) The work done by the force *F* and the magnitude of the force *F*,

b) If the frictional coefficient between the block and the inclined plane surface is 0.15, the speed of the block at the top of the inclined plane under the same force. (Use Work-Energy Theorem)



a) $\vec{F} = F_x\hat{i} + F_y\hat{j}$

$\boxed{\Delta E = E_s - E_i}$

$E_i = \frac{1}{2} m v_i^2$

$E_s = mgh + \frac{1}{2} m v_s^2$

$\Delta E = mgh + \frac{1}{2} m v_s^2 - \frac{1}{2} m v_i^2$

$\Delta E = 200.1{,}5 + \frac{1}{2}.20.4^2 - \frac{1}{2}.20.(0{,}5)^2$

$\boxed{\Delta E = 457{,}5\ J = W_F}$

$W_F = \vec{F}.\vec{d} = (F_x\hat{i} + F_y\hat{j}).d\hat{i} = F_x.d = F\cos 30°.d$

$457{,}5 = F.\cos 30°.3$

$\boxed{F = 176\ N}$

1 b) $\Sigma F_y = 0$

$N - F_y - mg\cos 30° = 0$

$N = mg\cos 30° + F.\sin 30°$

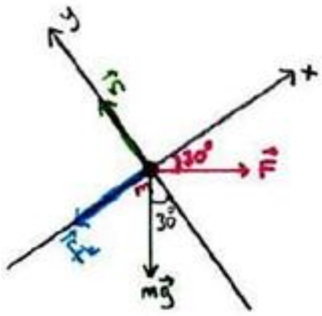
$N = 200.\cos 30° + 176.\sin 30°$

$\boxed{N = 261{,}2\ N}$

$f_k = \mu_k N$

$f_k = 0{,}15.261{,}2$

$\boxed{f_k = 39{,}18\ N}$



$W_F = 457{,}5\ J$

$W_{fk} = \vec{f_k}.\vec{d}$

$W_{fk} = f_k\, d \cos 180°$

$W_{fk} = -39{,}18.3 \cos 180°$

$W_{fk} = -117{,}54\ J$

$\Delta E = W_F + W_{fk} = \Delta K + \Delta U$

$W_F + W_{fk} = (\frac{1}{2}mv'^2 - \frac{1}{2}mv_i^2) + mgh$

$457{,}5 + (-117{,}54) = \frac{1}{2}.20.v'^2 - \frac{1}{2}.20.(0{,}5)^2 + 200.1{,}5$

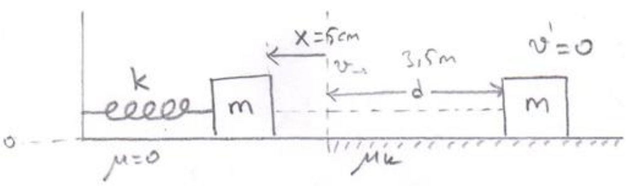
$\boxed{v' = 2{,}06\ m/s}$

2) A spring with spring constant $k = 200$ *N/m* is used as a launcher for a small block whose mass is 10 *g*. The block is placed against the compressed spring in a horizontal arrangement on a smooth horizontal surface. The spring, with the block, is compressed 5 *cm* and then released.

a) Find the speed of the block just as it leaves the spring,

b) The block encounters a rough surface as it leaves the spring. How much work does friction do in bringing the block to an eventual stop?

c) The block slides a distance of 3.5 *m* before stopping. What is the coefficient of kinetic friction between the block and surface?



a) $U_1 + \cancel{K_1}^{0} = \cancel{U_2}^{0} + K_2$

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$200.5^2 = 10.10^{-3}.v^2$

$\boxed{v = 7{,}1\ m/s}$

2 b) $W_{Total} = \Delta K$

$$W_{Total} = W_{fk} + \underbrace{W_g}_{0} + \underbrace{W_N}_{0} = \Delta K$$

$$W_{fk} = \Delta K = K' - K$$

$$W_{fk} = \frac{1}{2} m (\underbrace{v'^2}_{0} - v^2)$$

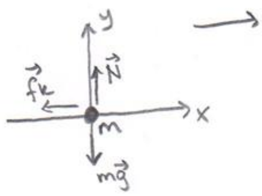
$$W_{fk} = -\frac{1}{2} 10 . 10^{-3} (7,1)^2$$

$$\boxed{W_{fk} = -0,25 J}$$

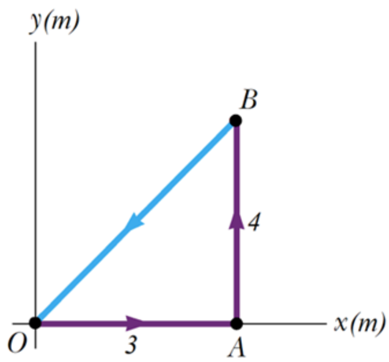
c) $W_{fk} = \vec{f_k} . \vec{d} = f_k . d . \cos 180° = - f_k d = -\mu_k N d = -\mu_k m g d$

$$-0,25 = -\mu_k . 10 . 10^{-3} . 9,8 . 3,5$$

$$\boxed{\mu_k = 0,73}$$



3) A particle of mass *m* moves in the *xy* plane under the action of force $\vec{F} = (4\hat{i} - 2\hat{j}) N$. Calculate the work done by the force as the particle moves in *OA*, *AB* and *BO*.



$$W = \vec{F} . \vec{r}$$

OA: $\vec{r} = (3\hat{i}) m$

$$W = (4\hat{i} - 2\hat{j}) . (3\hat{i}) J$$

$$W = 12 J$$

AB: $\vec{r} = (4\hat{j}) m$

$$W = (4\hat{i} - 2\hat{j}) . (4\hat{j}) J$$

$$W = -8 J$$

BO: $\vec{r} = (-3\hat{i} - 4\hat{j}) m$

$$W = (4\hat{i} - 2\hat{j}) . (-3\hat{i} - 4\hat{j}) J$$

$$W = -4 J$$

4) A force $\vec{F} = (4x\hat{i} + 3y\hat{j})\,N$ acts on a particle as the object moves in the x direction from the origin to $x = 5\,m$. Find the work done on the object by the force.

$$W = \int_{x_i}^{x_s} \vec{F} \cdot d\vec{r}$$

$$W = \int_0^5 (4x\hat{i} + 3y\hat{j}) \cdot dx\hat{i}$$

$$W = \int_0^5 4x\,dx = 4\frac{x^2}{2}\Big|_0^5$$

$$W = 50\,J$$

5) The restoring force for a spring that does not obey Hooke's law is $F(x) = -\alpha x - \beta x^2$, where $\alpha = 60$ *N/m*, $\beta = 18$ *N/m²* and the mass of spring can be negligible. Find the potential energy difference of the spring $U(x)$. (at $x = 0$; $U = 0$)

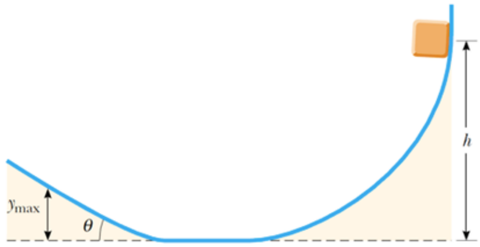
$$-\frac{du}{dx} = F_x = -\alpha x - \beta x^2$$

$$u = \int (\alpha x + \beta x^2)\,dx = \alpha\frac{x^2}{2} + \beta\frac{x^3}{3} + c$$

$$x = 0 \quad u = 0 \quad \Rightarrow \quad c = 0$$

$$u = 30x^2 + 6x^3 \text{ (joule)}$$

6) A block slides down a curved frictionless track and then up an inclined plane as in figure. The coefficient of kinetic friction between block and incline is μ_k. Use Work-Energy Theorem to find the maximum height ht reached by the block in terms of h, θ, μ_k.



$$mgy_{max} = mgh - \mu_k mgd\cos\theta \qquad d = \frac{y_{max}}{\sin\theta}$$

$$mgy_{max} = mgh - \mu_k mg\, y_{max} \cot\theta$$

$$y_{max} = \frac{h}{1+\mu_k \cot\theta}$$

7) A 50 kg trunk is pushed 6 m at constant speed up a 30° incline by a constant horizontal force (F). The coefficient of kinetic friction between the trunk and the incline is 0.2. What is the work done by;

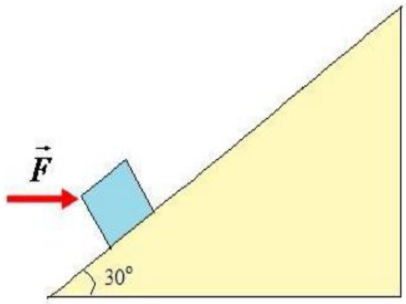
a) The applied horizontal force

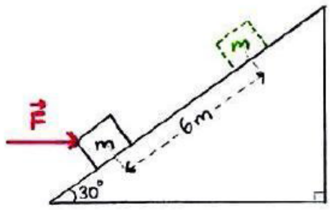
b) The force of friction

c) The force of gravity

d The normal force exerted by the incline

e) Determine the total work done on the trunk.



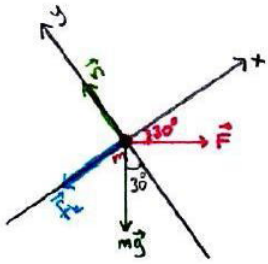


$$\sum \vec{F} = m\vec{a}$$

v= constant , a=0

$$\sum \vec{F} = 0$$

a)



$$\sum F_x = F\cos 30° - f_k - mg\sin 30° = 0$$

$$F\cos 30° - \mu_k . n - mg\sin 30° = 0 \qquad (1)$$

$$\sum F_y = n - F\sin 30° - mg\cos 30° = 0$$

$$n = F\sin 30° + mg\cos 30° \qquad (2)$$

$$F\cos 30° - \mu_k (F\sin 30° + mg\cos 30°) - mg\sin 30° = 0$$

$$F(\cos 30° - \mu_k \sin 30°) = mg(\sin 30° + \mu_k \cos 30°)$$

$$F = \frac{mg(\sin 30° + \mu_k \cos 30°)}{\cos 30° - \mu_k \sin 30°}$$

$m = 50\,kg$
$\mu_k = 0{,}2$
$g = 10\,m/s^2$

$$F = \frac{50.10\,(\sin 30° + 0{,}2\cos 30°)}{\cos 30° - 0{,}2.\sin 30°}$$

$$\boxed{F = 439{,}4 \text{ (N)}}$$

$$W_F = \vec{F}.\vec{d} = Fd\cos 30° = 439{,}4.6\cos 30°$$

$$\boxed{W_F = 2283{,}2 (J)}$$

b) $W_{fk} = \vec{f_k} \cdot \vec{d} = f_k d \cos 180°$

$W_{fk} = -\mu_k n d$ (2) → $n = F\sin30° + mg\cos30°$

$W_{fk} = -\mu (F\sin30° + mg\cos30°)\, d$

$W_{fk} = -0{,}2\,(439{,}4\ \sin30° + 50.10.\cos30°).6$

$\boxed{W_{fk} = -783{,}2\ (J)}$

c) $W_g = m\vec{g} \cdot \vec{d} = mgd\cos240°$

$W_g = 50.10.6.\cos240°$

$\boxed{W_g = -1500\ (J)}$

d) $W_n = \vec{n} \cdot \vec{d} = nd\cos90°$

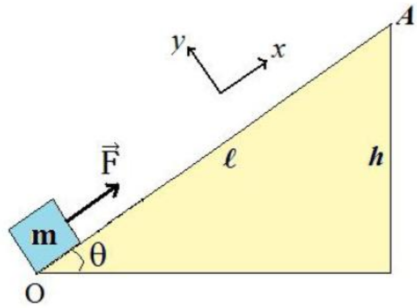
$\boxed{W_n = 0}$

e) $\sum W = W_F + W_{fk} + W_g + W_n$

$\sum W = 2283{,}2 - 783{,}2 - 1500 + 0$

$\boxed{\sum W = 0}$

8) A block of mass m is pushed at constant speed up an θ^0 incline by a constant horizontal force (**F**), from point O to point A through I= IOAI, as shown in Figure. The coefficient of kinetic friction between the block and the incline is changed with $\mu_k(x) = 0.1x$



a) Determine the work done by net force through IOAI

b) Draw a free-body diagram for the block and determine F(x) force in terms of m, g, θ and x.

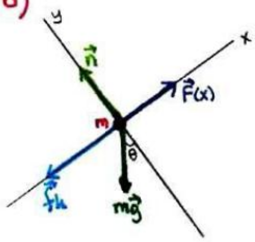
c) What is the work done by F(x) through IOAI in terms of m, g, θ and l.

a) $\sum \vec{F} = m\vec{a}$

$\sum \vec{F} = 0$ **(v=constant , a=0)**

$\boxed{W_{\sum\vec{F}} = 0}$

b)



$\sum F_x = F(x) - f_k - mg\sin\theta = 0$

$F(x) = mg\sin\theta + \mu_k n$ $\mu_k = 0{,}1x$

$F(x) = mg\sin\theta + 0{,}1 . n . x$ (1)

$\sum F_y = n - mg\cos\theta = 0$

$n = mg\cos\theta$ (2)

Using Eqns. (2) and (1); $\boxed{F(x) = mg\left[\sin\theta + 0{,}1\cos\theta\, x\right]}$

c) $W_F = \int_0^{\ell} F(x)\,dx$

$W_F = \int_0^{\ell} mg\left[\sin\theta + 0{,}1\cos\theta\, x\right]dx$

$W_F = \left[mg\sin\theta\, x + mg . 0{,}1\cos\theta \frac{x^2}{2} \right]_0^{\ell}$

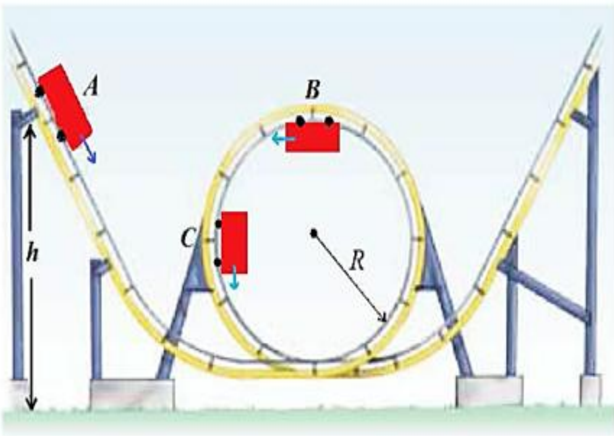
$W_F = mg\sin\theta\,\ell + 0{,}05\, mg\cos\theta\,\ell^2$

$\boxed{W_F = mg\ell\left[\sin\theta + 0{,}05 . \cos\theta . \ell\right]}$

9) A roller-coaster car is released from rest at the top of the first rise and then moves freely with negligible friction. The roller coaster shown in Figure has a circular loop of radius *R* in a vertical plane.

a) Find the required height of the incline in terms of R in order to avoid from dropping the car from the point B.

b) If h=(7/2)R and R=20 m, find the speed, centripetal acceleration and tangential acceleration of the car at point C.



a) $E_i = E_s$

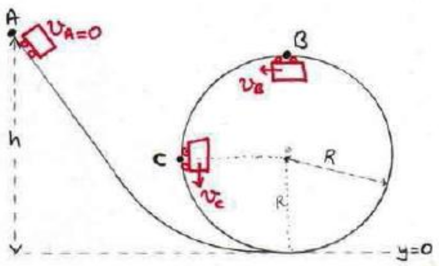
$K_A + \sum U_A = K_B + \sum U_B$ ($K_A = 0$)

$mgh_{min} = \frac{1}{2} m v_{B_{min}}^2 + 2mgR$

$mgh_{min} = \frac{1}{2} mgR + 2mgR$

$$\boxed{h_{min} = \frac{5}{2} R}$$

If $h \geq \frac{5}{2} R$, the roller-coaster will pass point B without falling.



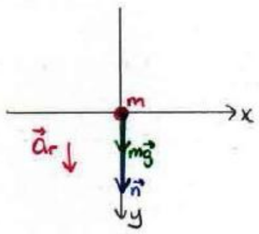
At point B

$\sum F_y = n + mg = ma_r$

$n \to 0$, $v_B \to min$

$mg = m \frac{v_{B_{min}}^2}{R}$

$v_{B_{min}}^2 = gR$



b) $E_A = E_C$

$K_A + \sum U_A = K_C + \sum U_C$ ($K_A = 0$)

$mgh = \frac{1}{2} m v_C^2 + mgR$

$h = \frac{7}{2} R$;

$mg \frac{7}{2} R = \frac{1}{2} m v_C^2 + mgR$

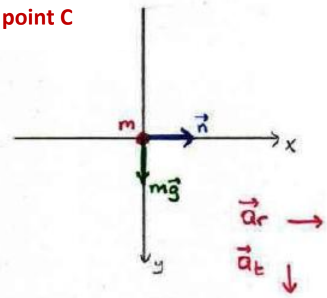
$\frac{5}{2} gR = \frac{1}{2} v_C^2$

$v_C = \sqrt{5gR}$

$v_C = \sqrt{5.10.20}$

$$\boxed{v_C = 31{,}6\ (m/s)}$$

At point C



$a_r = \frac{v_C^2}{R}$

$a_r = \frac{(31{,}6)^2}{20}$

$$\boxed{a_r = 50 (m/s^2)}$$

$a_t = g$

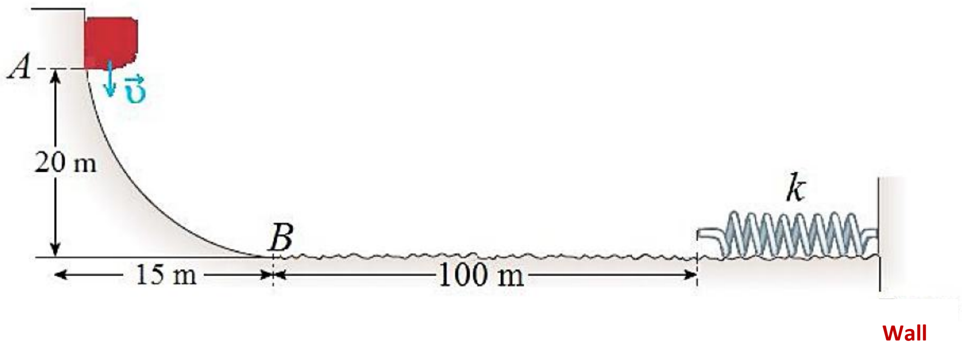
$$\boxed{a_t = 9{,}8 (m/s^2)}$$

10) A 15.0-kg block is released from point A with an initial speed 10.0 m/s as shown in Figure. The track is frictionless except for the portion between points B and spring , which has a length of 100.0 m. The block travels down the track, hits a spring of force constant 2 N/m. The coefficient of static friction and kinetic friction between the block and the surface is 0.8 and 0.2, respectively.

a) Find the speed of the block when it reaches the point B?

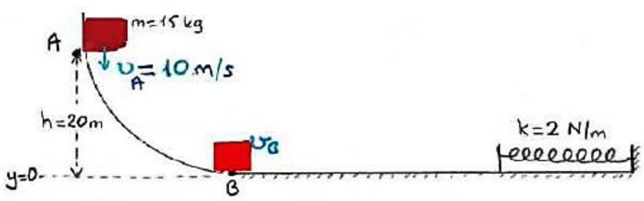
b) Determine the compression of the spring due to the block.

c) After stopped by spring, Whether the block moves again or not?



a)

There is no friction between points A and B



$$E_A = E_B$$

$$K_A + \sum U_A = K_B + \sum U_B$$

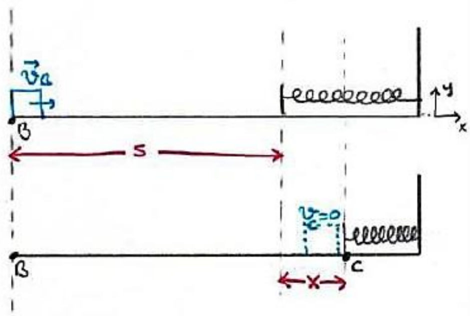
$$\frac{1}{2} m v_A^2 + mgh = \frac{1}{2} m v_B^2 + 0$$

$$\frac{1}{2} \cdot 15 \cdot 10^2 + 15 \cdot 10 \cdot 20 = \frac{1}{2} \cdot 15 \cdot v_B^2$$

$$\boxed{v_B = 22{,}36 \ (m/s)}$$

b)

The work done by all non-conservative forces is equal to the systems total mechanical energy change.



$$W_{ksuz} = E_s - E_i$$

$$W_{f_k} = (K_c + \sum U_c) - (K_B + \sum U_B)$$

$$-f_k (s+x) = \frac{1}{2} m v_c^2 + \frac{1}{2} k x^2 - \frac{1}{2} m v_B^2 - mgy$$

$$-\mu_k n (s+x) = 0 + \frac{1}{2} k x^2 - \frac{1}{2} m v_B^2 - 0$$

$$-0{,}2 \cdot 15 \cdot 10 \, (100 + x) = \frac{1}{2} \cdot 2 \cdot x^2 - \frac{1}{2} \cdot 15 \cdot (22{,}36)^2$$

$x^2 + 30x + 3000 - 3750 = 0$

$x^2 + 30x - 750 = 0$

$\boxed{x = 16{,}2 (m)}$

$\Delta = b^2 - 4ac$

$\Delta = 30^2 - 4.1.(-750)$

$\Delta = 3900$

$x_{1,2} = \dfrac{-b \mp \sqrt{\Delta}}{2a}$

$x_{1,2} = \dfrac{-30 \mp \sqrt{3900}}{2}$

$x_1 = -46{,}2 \qquad x_2 = 16{,}2$

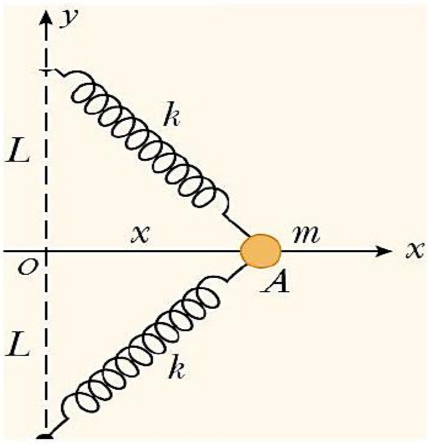
c) **In order to move again, the force exerted by the spring on the block must be greater than the static friction force.**

$f_s = \mu_s . n = \mu_s . mg = 0{,}8 . 15 . 10 = 120 (N)$

$F_y = kx = 2 . 16{,}2 = 32{,}4 (N)$

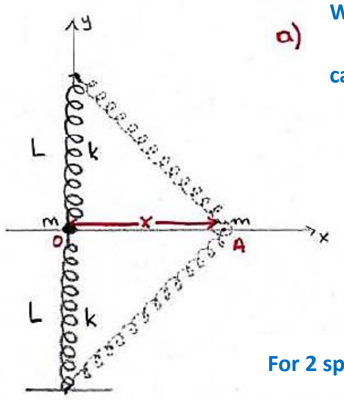
$F_y < f_s$ **The block can not move.**

11) A particle is attached between two identical springs on a horizontal frictionless table. Both springs have spring constant k and are initially unstressed. If the particle is pulled a distance x=3 m along a direction perpendicular to the initial configuration of the springs, as in Figure;



a) What is its speed when it comes back the equilibrium point *x*=0?

b) Find its acceleration at the instant that it is released from point A (k= 40 N/m, m=8 kg and L=4m).



a) **When the mass moves distance x, the length of each spring changes from L to $\sqrt{x^2+L^2}$, so each exerts force $k\left(\sqrt{x^2+L^2}-L\right)$ towards its fixed end. The y-components cancel out.**

$$U(x) = \frac{1}{2}k\left(\sqrt{x^2+L^2}-L\right)^2$$

$$U(x) = \frac{1}{2}k\left[(x^2+L^2) - 2L\sqrt{x^2+L^2} + L^2\right]$$

$$U(x) = \frac{1}{2}kx^2 + kL\left(L - \sqrt{x^2+L^2}\right)$$

For 2 spring:

$$\boxed{U(x) = kx^2 + 2kL\left(L - \sqrt{x^2+L^2}\right)}$$

$k = 40$ N/m
$m = 8$ kg
$L = 4$ m
$x = 3$ m

$$U = 40 \cdot 3^2 + 2 \cdot 40 \cdot 4\left(4 - \sqrt{3^2+4^2}\right) = U_i$$

$$\underline{U = 40 (J)}$$

$$\Delta K = -\Delta U$$

$$K_s - \underbrace{K_i}_{0} = -(\underbrace{U_s}_{0} - U_i)$$

$$\frac{1}{2}mv^2 = U_i$$

$$\frac{1}{2} \cdot 8 \cdot v^2 = 40 \quad , \quad \boxed{v \cong 3,2 \ (m/s)}$$

b) $$\vec{F}(x) = -\frac{dU(x)}{dt}$$

$$\vec{F}(x) = -\frac{d}{dt}\left[kx^2 + 2kL\left(L - \sqrt{x^2+L^2}\right)\right]\hat{i}$$

$$\vec{F}(x) = \left(-2kx + \frac{2kLx}{\sqrt{x^2+L^2}}\right)\hat{i}$$

$$\vec{a} = \frac{\vec{F}}{m}$$

$$\vec{a} = \left(-\frac{2 \cdot 40 \cdot 3}{8} + \frac{2 \cdot 40 \cdot 4 \cdot 3}{8 \cdot \sqrt{3^2+4^2}}\right)\hat{i} \quad , \quad \boxed{\vec{a} = -6\,\hat{i} \ (m/s^2)}$$