Center of Mass

1) Four objects are situated along the *y* axis as follows: a 2.00 kg object is at +3.00 m, a 3.00-kg object is at +2.50 m, a 2.50-kg object is at the origin, and a 4.00-kg object is at -0.500 m. Where is the center of mass of these objects?

The *x*-coordinate of the center of mass is

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0+0+0+0}{(2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg})}$$

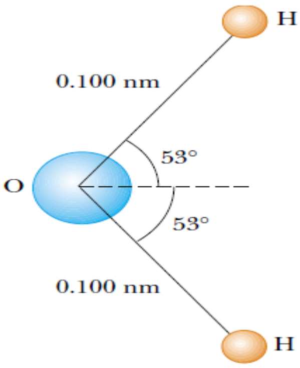
$$\boxed{x_{CM} = 0}$$

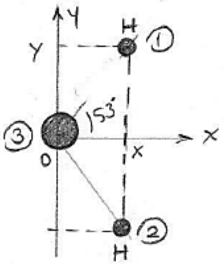
and the *y*-coordinate of the center of mass is

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(2.00 \text{ kg})(3.00 \text{ m}) + (3.00 \text{ kg})(2.50 \text{ m}) + (2.50 \text{ kg})(0) + (4.00 \text{ kg})(-0.500 \text{ m})}{2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg}}$$

$$\boxed{y_{CM} = 1.00 \text{ m}}$$

2) A water molecule consists of an oxygen atom with two hydrogen atoms bound to it. The angle between the two bonds is 106°. If the bonds are 0.100 nm long, where is the center of mass of the molecule?





$X = 0,1 \cdot \cos 53 = 0,06 \text{ nm}$

$Y = 0,1 \cdot \sin 53 = 0,08 \text{ nm}$

$m^H = 1,008 \text{ akb}$

$m^O = 15,999 \text{ akb}$

$\vec{r}_1 = x \cdot \vec{i} + y\vec{j} = 0,06\vec{i} + 0,08\vec{j} \text{ nm}$

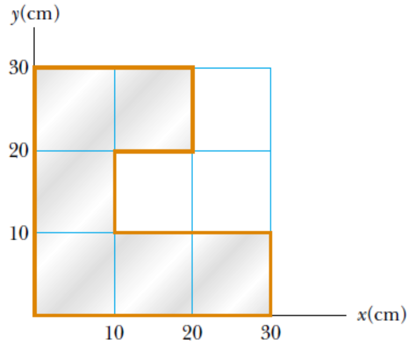
$\vec{r}_2 = 0,06\vec{i} - 0,08\vec{j}$

$\vec{r}_3 = 0$

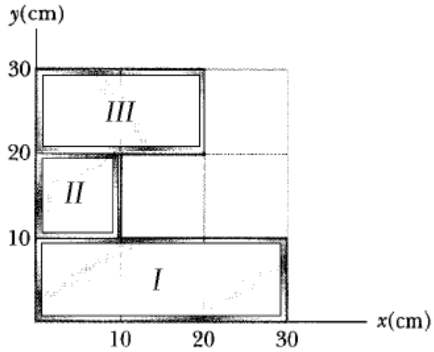
$$\vec{r}_{km} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1+m_2+m_3} = \frac{1,008(0,06\vec{i} + \cancel{0,08\vec{j}} + 0,06\vec{i} - \cancel{0,08\vec{j}})}{2(1,008) + 15,999}$$

$$= \frac{1,008 \cdot (0,12)}{18,015} = \underline{6,71 \cdot 10^{-3}}\, \vec{i} \text{ (nm)}$$

3) A uniform piece of sheet steel is shaped as in Figure. Compute the *x* and *y* coordinates of the center of mass of the piece.



Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.



$$A = A_1 + A_2 + A_3$$
$$M = M_1 + M_2 + M_3$$
$$\frac{M_1}{A_1} = \frac{M}{A}$$
$$A_1 = 300\text{ cm}^2,\ A_2 = 100\text{ cm}^2,\ A_3 = 200\text{ cm}^2,\ A = 600\text{ cm}^2$$
$$M_1 = M\left(\frac{A_1}{A}\right) = \frac{300\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{2}$$
$$M_2 = M\left(\frac{A_2}{A}\right) = \frac{100\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{6}$$
$$M_3 = M\left(\frac{A_3}{A}\right) = \frac{200\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{3}$$
$$x_{CM} = \frac{x_1M_1 + x_2M_2 + x_3M_3}{M} = \frac{15.0\text{ cm}\left(\frac{1}{2}M\right) + 5.00\text{ cm}\left(\frac{1}{6}M\right) + 10.0\text{ cm}\left(\frac{1}{3}M\right)}{M}$$
$$x_{CM} = \boxed{11.7\text{ cm}}$$
$$y_{CM} = \frac{\frac{1}{2}M(5.00\text{ cm}) + \frac{1}{6}M(15.0\text{ cm}) + \left(\frac{1}{3}M\right)(25.0\text{ cm})}{M} = 13.3\text{ cm}$$
$$y_{CM} = \boxed{13.3\text{ cm}}$$

4) A rod of length 30.0 cm has linear density (mass-perlength) given by

$$\lambda = 50.0\ \mathrm{g/m} + 20.0x\ \mathrm{g/m^2},$$

where *x* is the distance from one end, measured in meters.

(a) What is the mass of the rod? (b) How far from the *x* = 0 end is its center of mass?

a) $M = \int dm = \int_0^{0,3} \lambda\, dx = \int_0^{0,3} \left[50 + 20x\right] dx$

$M = \left[50x + 20\frac{x^2}{2}\right]_0^{0,3} = 15,9\ g.$

b) $x_{km} = \frac{1}{M}\int x\, dm = \frac{1}{M}\int_0^{0,3} x\,(\lambda\, dx)$

$x_{km} = \frac{1}{M}\int_0^{0,3} x\,(50 + 20x)\, dx = \frac{1}{M}\int_0^{0,3}(50x + 20x^2)\, dx$

$x_{km} = \frac{1}{15,9}\left[50\frac{x^2}{2} + 20\frac{x^3}{3}\right]_0^{0,3} = 0,153\ m.$