# Experiment 9

## SIMPLE PENDULUM AND SPRING PENDULUM

**Aims:**

i) Determination of a spring constant and observation of the period of an object in simple harmonic motion.
ii) Determination of gravitional acceleration by simple pendulum.

**Equipments:** Spring, strings with different lenghts, pendulum bobs (spheres) and mases, ruler, stopwatch, graphic paper, scientific calculator.

### 1. Introduction

**Simple Harmonic Motion:** A motion which is reperated with certain time intervals is called periodic motion, when an object has a periodic motion around a fixed spot, it is called oscillation.

Harmonic motion is a periodic motion in the form of sinus or cosinus function. When the force acting on an object having harmonic motion is balanced its position is called equalibrium position and the distance from the eqilibrium position in any time is called displacement. If the restoring force acting on the particle to equilibrium position is proportional to displacement, the motion is called simple harmonic motion.

If an object is pulled away and then released, it makes simple harmonic motion. The direction of the restoring force and the displacement are opposite, so that,

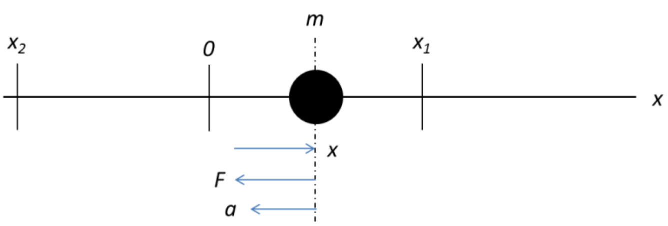



**Figure- 1.** The object in simple harmonic motion.

$$F = -kx \tag{1}$$

where k is the constant. On the other hand, in the terms on Newton's second law, this restoring force is,

$$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2} \tag{2}$$

thus,

$$-kx = m\frac{dv}{dt} \quad \text{or} \quad m\frac{d^2y}{dt^2} + kx = 0 \tag{3}$$

If, $\omega^2 = k/m$ ($\omega$; angular frequency ) the last equation becomes,

$$\frac{d^2y}{dt^2} + \omega^2 x = 0 \tag{4}$$

Equation (4) is called harmonic oscillator equation and its solution is,

$$y = Asin(\omega t + \delta) \tag{5}$$

where A is the amplitude and δ is the initial phase.

with the help of equation (5) we gain;

$$v = \frac{d\ x}{dt} = \omega A cos(\omega t + \delta) \tag{6}$$

$$a = \frac{d\ v}{dt} = -\omega^2 A sin(\omega t + \delta) = -\omega^2 y \tag{7}$$

The angular frequency can be written as, $\omega = \frac{2\pi}{T}$ , then the period of the simple harmonic motion is,

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{8}$$

**Gravitional Acceleration:** When an object is released from a certain height it falls with speeding up. As the object does not have an initial velocity, a force is necessary to start the motion. This can be explained with the laws of dynamics as it gains an acceleration. On the other hand, an object having a free fall is speeding up, it is obvious that it has an acceleration. This acceleration acting on the objects is called, gravitional acceleration (g), the force acting on the object is called the weight of the object. If m is the mass of the object,

$$G = mg \tag{9}$$

In other words, G is the force acting on the force by the earth and it is usually called as gravitional force. Depending on the Newton's third law, as the earth act on the object with a force of G, the object acts a force on the earth as a response.

**Simple Pendulum:** The system with a fixed light string carrying ana mass is called simple pendulum (Figure-2). If the mass is pulled over from the equilibrium position and then released, it will make perdiodic oscillations along the vertical axis with the mg gravitional force and under the T tension on the string. As shown in Figure-1, on the (*x, y*) plane, the component of $mg$ along the x axis is $mg\sin\theta$, the component along the *y* axis is $mg\cos\theta$. Thus the tension on the string T is balanced by $mg\cos\theta$

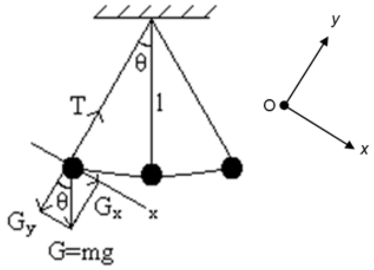



**Figure- 2**

$mg\sin\theta$ component is the intensity of the restoring force and can be written as,

$$F = mgsin\theta \tag{10}$$

If the angle $\theta$ is small (<5°), $\sin\theta \approx \theta$ so, $\theta = x/\ell$ . now the restoring force is,

$$F = \text{-}mg\theta = \text{-}mg\frac{x}{\ell} \tag{11}$$

Thus, the restoring force is directly proportional to displacement for small displacement values ($\mathrm{F}\ \alpha\ \mathrm{x}$). So the simple pendulum makes simple harmonic motion. According to this, the equation below can be written,

$$F = \text{-}kx \tag{12}$$

Where k is the ratio factor. The (-) sign in equation (12) means it is the restoring force. Using equations (11) and (12),

$$-kx = -mg\frac{x}{\ell} \quad \text{veya} \quad k = \frac{mg}{\ell} \tag{13}$$

Using Newton's second law given by, $F = m\left(\frac{d^2x}{dt^2}\right)$, we can write,

$$-kx = m\left(\frac{d^2x}{dt^2}\right) \tag{14}$$

using $\omega^2 = \frac{k}{m}$, equation (13) becomes

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{15}$$

This is the differantial equation of simple harmonic motion. The solution of equation (15) is given below,

$$x = A\sin(\omega t + \delta) \tag{16}$$

where A is the constant amplitude value and δ is the initial phase. The solution as a function of the inital condition can be written as,

$$x = A\cos(\omega t + \delta) \tag{16a}$$

On the other hand, as $\omega = \frac{2\pi}{T}$ , the period of the motion is,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/\ell}} = 2\pi\sqrt{\frac{\ell}{g}} \tag{17}$$

One can understand that for small oscillations, the period of the simple pendulum is not dependent to the mass of the pendulum bob and the amplitude, it just depends on the length of the pendulum and the gravitional acceleration.

*"Note that equation (17) is valid if only the angle $\theta$ is small enough."*

**2. Experiment**

**Determination of Spring Constant**

1. Hang the mass m at the end of the spring and pull it down softly from the equilibrium position and then release the system. Observe the simple harmonic motion of the system around the equilibrium position.

2. Measure the time for 10 periods to determine the period of the simple harmonic motion. Calculate the average period and write the values down in Table-1.

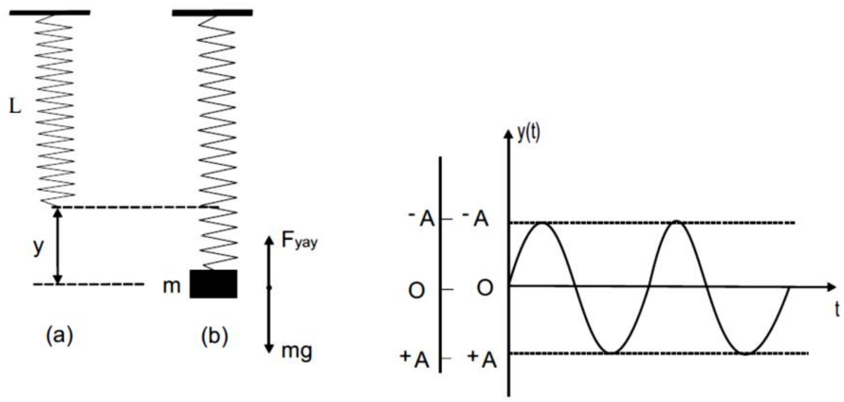


**Figure 4.** Spring pendulum. The spring lenghtens $\Delta y$ when weight m is hanged. When the spring force equals the weight of the mass, the system is in equilibrium. If the mass is pulled down $y = A$ from the equilibrium position and then released, simple harmonic motion will be observed.

3. Repeat the experiment by adding extra masses on the spring and calculate the average periods for each mass values. Write the values down in Table-1.

**Table- 1**

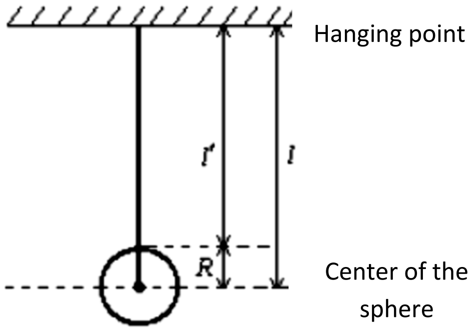
| $m$ (kg) | $10T$ (s) | $T_{ave}$ (s) | $T^2$ ($s^2$) | $k$ (N/m) |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

4. Plot the $T^2 = f(m)$ graph using the values from Table-1. This is supposed to be a straight line passing through the origin point.

5. Calculate the spring constant by equation (8) and choosing two $(m,\ T^2)$ pairs. Write down the results in Table-1.

**Determination of Gravitional Acceleration**

6. Measure the length of the string from the hanging point to the sphere $\ell'$ , and the diameter of the sphere by a caliper and calculate the length of the pendulum $\ell = \ell' + R$. Repeat this process for the strings with 4 different legths and write down the values in Table-2.




**Şekil 5**

7. Pull over the pendulum a little (approximately 5°) and let it oscillate. Measure the time for 10 periods of oscillation by stopwatch and find the period of the pendulum. (Note that 1 period is the time interval passing, as the pendulum reaches the point it starts the motion.)

8. Repeat the experiment with strings with different lengths (at least 4 times) and write down the values in Table-2.

$2R = 10cm$ $\quad R = 5cm$ $\quad \ell'$ = ..... $\quad g_{theoretical} = 9{,}8\,m/s^2$

**Table-2**

| $\ell = \ell' + R$ (m) | 10T (s) | $T_{ave}$ (s) | $g_{ave}$ ($m/s^2$) | $\frac{\lvert\Delta g\rvert}{g_{theoretical}}$ |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

9. Plot the $T^2 = f(\ell)$ graph with the help of Table-2. Calculate the gravitional acceleration g by the $\ell/T^2$ ratio from the graph and equation (17).

10. Calculate the relative error on determining the gravitional acceleration comparing the theoretical value from the equation below and write down in Table-2.

$$\frac{\lvert\Delta g\rvert}{g_{theoretical}} = \frac{\lvert g_{theoretical} - g_{ave}\rvert}{g_{theoretical}} \tag{18}$$

**Bu sayfa boş bırakılmıştır.**